Practical trigonometry - Practice test 3: connected problems

10 questions | 52 marks | 70 minutes

Use degree mode. Give calculated lengths and angles to one decimal place unless stated otherwise; bearings use three digits to the nearest degree. In triangle ABC, a is opposite A, b opposite B and c opposite C. Show working and units.

Equivalent correct methods earn the corresponding marks.

1. Similar displays have length ratio small:large=2:3. The small display has a side 9 cm and area 28 cm². [4 marks]

(a) Find the matching large side. [2 marks]

Length factor=3/2.

9×1.5=13.5.

Answer: 13.5 cm

(b) Find the large area. [2 marks]

Area factor=(3/2)²=9/4.

28×9/4=63.

Answer: 63 cm²

2. Triangle ABC is right-angled at C, with AC=15 cm and AB=17 cm. [5 marks]

(a) Find BC and sinA. [3 marks]

BC=√(17²−15²)=8.

sinA=BC/AB=8/17.

Answer: BC=8 cm; sinA=8/17.

(b) Find cosB and explain its relationship to sinA. [2 marks]

A and B are complementary.

BC is adjacent to B but opposite A.

Answer: cosB=8/17=sinA.

3. A 13 m ladder slides on level ground against a vertical wall. Its foot moves from 5 m to 12 m from the wall. [4 marks]

(a) Find the original and final heights of its top. [2 marks]

Original=√(169−25)=12.

Final=√(169−144)=5.

Answer: 12 m and 5 m.

(b) Find the vertical drop. [2 marks]

Drop=12−5=7 m, not the final height alone.

Answer: 7 m

4. A right triangle has opposite leg 5 cm and adjacent leg 12 cm relative to θ. [4 marks]

(a) Find both acute angles. [2 marks]

$\theta = \tan^{-1}(5/12)$.

The other is $90° - \theta$.

Answer: 22.6° and 67.4°.

(b) Explain the error in using $\sin^{-1}(5/12)$. [2 marks]

The hypotenuse is 13.

Sine would use 5/13, whereas tangent uses 5/12.

Answer: 12 is adjacent, not the hypotenuse.

5. Two ground-level observers and a tower base lie on one straight level line. Both observers are on the same side of the tower, 20 m apart. The nearer elevation is 60°, the farther 30°. [7 marks]

(a) Let x be the nearer distance. Form an equation. [3 marks]

Both sides equal the tower height.

Answer: $x \tan 60° = (x+20)\tan 30°$.

(b) Find the two distances and the tower height. [4 marks]

Solve the equation to obtain $x = 10$.

$h = 10\tan 60° = 10\sqrt{3}$.

Answer: 10 m and 30 m; height 17.3 m.

6. A boat travels 10 km on bearing 030°, then 15 km due east. [6 marks]

(a) Find its final eastward and northward distances. [3 marks]

First east component=10sin30°=5.

Final east=5+15=20; north=10cos30°.

Answer: East 20 km; north 8.7 km.

(b) Find its direct return distance and bearing. [3 marks]

Distance=$\sqrt{(20^2+(10\cos 30°)^2)}$.

Find the outward bearing from north, then add 180°.

Answer: 21.8 km; 247°.

7. A 10 m diagonal divides a quadrilateral into two triangles. The perpendicular heights from the other vertices are 4 m and 7 m, on opposite sides of the diagonal. [4 marks]

(a) Find the two triangle areas. [2 marks]

½×10×4=20; ½×10×7=35.

Answer: 20 m² and 35 m².

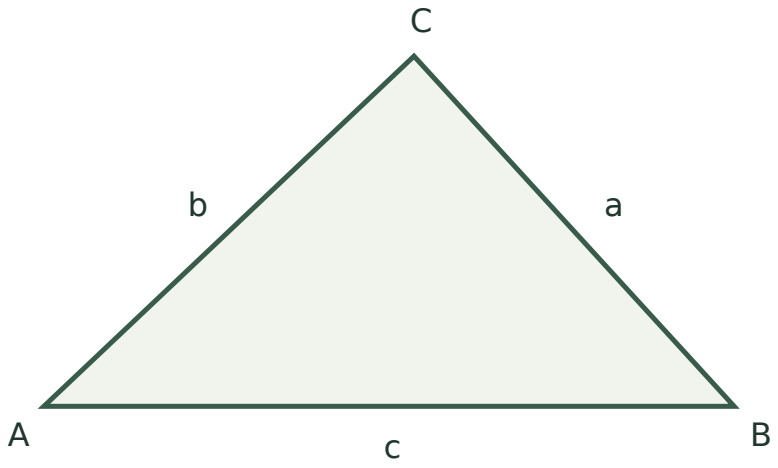
(b) Find total area and explain why an outside sloping side cannot replace either height. [2 marks]

Add 20+35.

A height must be perpendicular to the selected base.

Answer: 55 m²

8. Triangle ABC has A=30°, a=10 cm and b=14 cm. [7 marks]



Sketch not to scale

(a) Find every possible pair B,C. [4 marks]

sinB=0.7; take both sine candidates.

C=150°−B.

Answer: B=44.4°, C=105.6°; or B=135.6°, C=14.4°.

(b) Find c for each triangle. [3 marks]

a/sinA=20.

Use c=20sinC for each C.

Answer: 19.3 cm and 5.0 cm.

9. Sides 8 cm and 13 cm enclose 45°. [5 marks]

(a) Find the third side. [2 marks]

$c^2=64+169-208\cos 45°$.

Answer: 9.3 cm

(b) Find area and explain why these two sides are not Pythagoras legs. [3 marks]

$K=\frac{1}{2}\times 8\times 13\sin 45°$.

The sides enclose 45°, not a right angle.

Answer: 36.8 cm^2

10. A survey triangle is proposed with A=45°, a=6 m and b=10 m. [6 marks]

(a) Show that no triangle fits. [3 marks]

$\sin B=10\sin 45°/6\approx 1.1785$.

Answer: $\sin B\approx 1.179>1$; impossible.

(b) Find the smallest possible a for A=45°, b=10 m. Describe the boundary triangle. [3 marks]

Require $10\sin 45°/a\leq 1$.

At equality $\sin B=1$, giving exactly one right triangle.

Answer: 7.1 m; B=90°.