Practical trigonometry - Practice test 5: applications and reasoning

10 questions | 58 marks | 75 minutes

Use degree mode. Give calculated lengths and angles to one decimal place unless stated otherwise; bearings use three digits to the nearest degree. In triangle ABC, a is opposite A, b opposite B and c opposite C. Show working, units and reasons for rejected solutions.

Equivalent correct methods earn the corresponding marks.

1. Similar signs have area ratio small:large=25:81. The smaller sign has a side 7.5 cm and perimeter 22.5 cm. [5 marks]

(a) Find the length scale factor and matching large side. [3 marks]

The length factor is √(81/25)=9/5.

7.5×9/5=13.5.

Answer: 9/5; 13.5 cm.

(b) Find the larger perimeter. [2 marks]

Perimeter scales linearly: 22.5×9/5=40.5.

Answer: 40.5 cm

2. A right triangle has perpendicular legs 20 cm and 21 cm. [4 marks]

(a) Find the hypotenuse. [2 marks]

$c^2=400+441=841$.

c=29.

Answer: 29 cm

(b) Find its two acute angles. [2 marks]

Use tanθ=20/21.

The second acute angle is the complement.

Answer: 43.6° and 46.4°.

3. A cable runs from a roof point 12 m above level ground to an anchor 20 m horizontally from the building. [6 marks]

(a) Find its length and its angle above horizontal. [3 marks]

$L=\sqrt{(12^2+20^2)}$.

$\theta=\tan^{-1}(12/20)$.

Answer: 23.3 m; 31.0°.

(b) The anchor moves 2 m farther away on the same straight ground line. How much longer must the cable become? [3 marks]

New length$=\sqrt{(12^2+22^2)}$.

Subtract the original unrounded length.

Answer: 1.7 m

4. An acute angle θ satisfies $\sin\theta=\cos38°$. [4 marks]

(a) Find θ and explain without trial and error. [2 marks]

For complementary acute angles, $\sin\theta=\cos(90°-\theta)$.

$\theta=90°-38°=52°$.

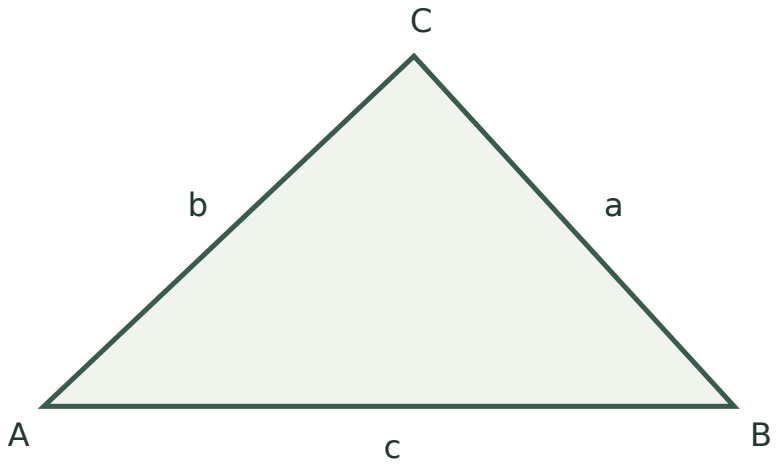
Answer: 52°

(b) Find $\tan\theta$ to three decimal places. [2 marks]

Evaluate $\tan52°$ in degree mode.

Answer: 1.280

5. Triangle ABC has A=40°, a=9 cm and b=12 cm. [7 marks]



Sketch not to scale

(a) Find all possible pairs B,C. [4 marks]

sinB=12sin40°/9.

Use the acute candidate and its supplement; C=140°−B.

Answer: B=59.0°, C=81.0°; or B=121.0°, C=19.0°.

(b) Find c for each possible triangle. [3 marks]

For each C, use c=9sinC/sin40°.

Keep the two sets of angles and sides paired.

Answer: 13.8 cm and 4.6 cm.

6. A boat travels 10 km on bearing 060°, then 12 km on bearing 150°. [7 marks]

(a) Find its direct distance from the start. [3 marks]

The travel vectors are perpendicular because $150° - 60° = 90°$.

$d = \sqrt{(10^2 + 12^2)}$.

Answer: 15.6 km

(b) Find the direct return bearing. [4 marks]

$E = 10\sin 60° + 12\sin 150° \approx 14.7$; $N = 10\cos 60° + 12\cos 150° \approx -5.4$.

The endpoint is south-east. Use the correct quadrant for the outward bearing, then reverse it.

Answer: 290°

7. A triangular panel has area 24 cm². Two sides are 8 cm and x cm, with included angle 30°. [4 marks]

(a) Find x. [2 marks]

$24 = \frac{1}{2} \times 8 \times x \times \sin 30° = 2x$.

Thus $x = 12$.

Answer: 12 cm

(b) With those same two sides, find another included angle giving the same area. [2 marks]

The supplement has the same sine: $\sin 150° = \sin 30°$.

Answer: 150°

8. A triangular frame has sides 9 cm, 10 cm and 15 cm. [6 marks]

(a) Find its largest angle and state whether it is acute or obtuse. [3 marks]

The largest angle faces side 15.

$\cos C = (81+100-225)/(2\times 9\times 10) = -44/180$.

Answer: 104.1°; obtuse.

(b) Calculate its area. [3 marks]

The angle is included between sides 9 and 10.

$K = \tfrac{1}{2}\times 9\times 10\sin C$.

Answer: 43.6 cm²

9. An instrument records A=65°, a=9 m and b=14 m for a proposed triangle. [6 marks]

(a) Demonstrate why the readings cannot all be correct. [3 marks]

$\sin B = 14\sin 65°/9 \approx 1.410$.

No angle has sine greater than 1.

Answer: $\sin B \approx 1.410 > 1$.

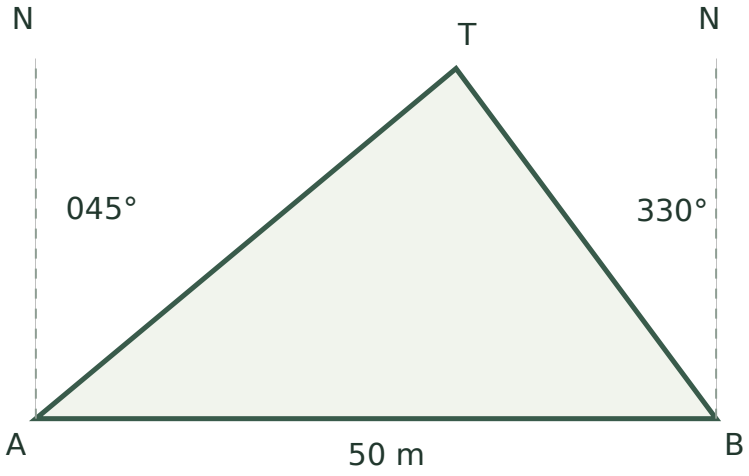
(b) If A and b are correct, find the smallest possible a. Describe the limiting triangle. [3 marks]

a must be at least $b\sin A = 14\sin 65°$.

At equality B=90°; hence C=25°.

Answer: 12.7 m; B=90°, C=25°.

10. On level ground, B is 50 m due east of A. A marker T is seen from A on bearing 045° and from B on bearing 330°. [9 marks]



Sketch not to scale

(a) Find all three interior angles of triangle ABT. [3 marks]

At A, angle between east090° and T045° is 45°.

At B, BA has bearing270° and BT330°, giving60°.

T=180°−45°−60°=75°.

Answer: A=45°, B=60°, T=75°.

(b) Find AT and BT. [3 marks]

Use baseline50 opposite angle75°.

AT=50sin60°/sin75°; BT=50sin45°/sin75°.

Answer: AT=44.8 m; BT=36.6 m.

(c) Find perpendicular distance from T to AB, then the area of ABT. [3 marks]

h=ATsin45°.

K=½×50×h, retaining precision until the end.

Answer: Height 31.7 m; area 792.5 m².