Practice test 2 - Reading representations

10 questions | 53 marks | 60 minutes

Interpret equations, intervals, tables and financial comparisons, then justify predictions.

Equivalent correct methods earn the corresponding marks.

1. Use algebra before making a practical calculation. [4 marks]

(a) Solve $2(3x - 4) = 5x + 9$. [2 marks]

Expand: $6x - 8 = 5x + 9$.

Subtract $5x$ and add 8.

Answer: $x = 17$.

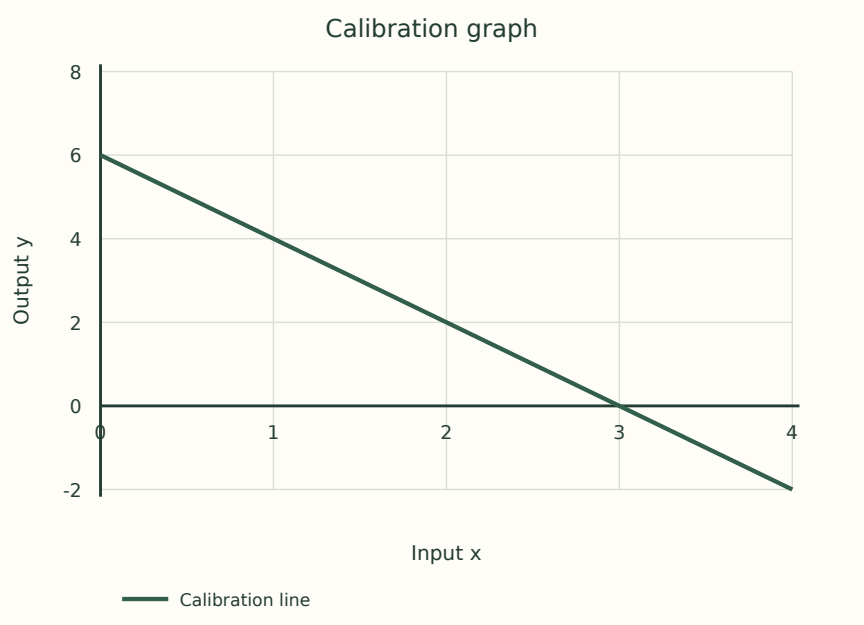
(b) The formula $A = (a + b)h/2$ has $A = 54$, $a = 4$ and $b = 8$. Find h. [2 marks]

$54 = 12h/2 = 6h$.

Divide by 6.

Answer: $h = 9$.

2. Use the calibration graph below. Horizontal grid steps represent 1 input unit and vertical grid steps represent 2 output units. [6 marks]



(a) Read the two axis intercepts. [2 marks]

Locate where the plotted line crosses each labelled axis.

Answer: Y-intercept $(0, 6)$; x-intercept $(3, 0)$.

(b) Use two points on the graph to find the gradient and write the rule. [2 marks]

Using the intercepts, $m = (0 - 6)/(3 - 0) = -2$.

The y-intercept is 6.

Answer: Gradient -2; $y = -2x + 6$.

(c) Would $y = -2x + 9$ be parallel to the graphed line? Explain. [2 marks]

The graph’s slope is -2; the proposed rule has the same coefficient of x.

Answer: Yes; both have gradient -2 and different intercepts.

3. A cost table gives $(t, C) = (1, 12), (3, 20), (5, 28)$, with t in hours and C in dollars. [6 marks]

(a) Construct a linear cost rule. [2 marks]

Rate $= (20 - 12)/(3 - 1) = 4$.

Use $12 = 4 + b$ to find $b = 8$.

Answer: $C = 4t + 8$.

(b) Interpret both parameters. [2 marks]

The gradient has units dollars/hour.

The intercept is cost at zero hours in the rule.

Answer: 4 dollars per hour; an 8-dollar fixed cost.

(c) Predict C at t = 4 and classify the prediction. [2 marks]

$4(4) + 8 = 24$.

4 lies between observed inputs 1 and 5.

Answer: 24 dollars; interpolation.

4. Plumber A charges $80 + 50h$ dollars and B charges $140 + 35h$ dollars. [6 marks]

(a) Find the equal-cost time and cost. [2 marks]

$80 + 50h = 140 + 35h$ gives $15h = 60$.

Substitute h = 4.

Answer: 4 h; 280 dollars.

(b) Which is cheaper for 2 h? [2 marks]

Evaluate both at h = 2.

Answer: A: 180 dollars, compared with B: 210 dollars.

(c) What is the first whole number of hours for which B is strictly cheaper? [2 marks]

B is cheaper for $h > 4$.

At 4 their costs are equal.

Answer: 5 h.

5. Postage is 3 dollars for $0 < m \leq 250$ g, 5 dollars for $250 < m \leq 500$ g and 8 dollars for $500 < m \leq 1000$ g. [4 marks]

(a) Find charges for 400 g, 500 g and 501 g. [2 marks]

400 and 500 belong to the middle band.

501 belongs to the final band.

Answer: 5, 5 and 8 dollars respectively.

(b) Is the fee continuous at 500 g? Explain. [2 marks]

The charge is constant in each band, not proportional to mass.

Answer: No; it jumps from 5 to 8 dollars just above 500 g.

6. Consecutive values of a quantity are 100,90,81,72.9 starting at n = 0. [4 marks]

(a) Write an exponential rule and state the rate. [2 marks]

Every ratio is 0.9.

The removed proportion is 0.1.

Answer: $P = 100(0.9)^n$; 10% decay per step.

(b) Predict the value at n = 4. [2 marks]

$100(0.9)^4 = 65.61$.

Answer: 65.61.

7. A quantity grows from 200 to 242 over two equal steps. [5 marks]

(a) Find the positive multiplying factor and percentage growth per step. [2 marks]

$b^2 = 242/200 = 1.21$.

Thus b = 1.1.

Answer: 1.1; 10%.

(b) Find the first whole step at which the model exceeds 300. [3 marks]

At 4: $200(1.1)^4 = 292.82$.

At 5: $200(1.1)^5 = 322.102$.

Answer: Step 5.

8. A population begins at 60 and doubles every 3 years. Let t be years after the start. [6 marks]

(a) Write a model using t. [2 marks]

The exponent counts three-year intervals.

Answer: $P = 60 \times 2^{t/3}$.

(b) Find the population predicted after 6 years. [2 marks]

Six years is two doubling intervals: $60(2)^2 = 240$.

Answer: 240.

(c) Find the annual multiplying factor, to four decimal places. [2 marks]

One year is one-third of a doubling interval.

The factor is $2^{1/3} \approx 1.2599$.

Answer: About 1.2599.

9. At t = 1 and t = 3, an exponential quantity is 50 and 98. A new value at t = 4 is 142. [6 marks]

(a) Find the positive step factor and a model. [2 marks]

The two-step ratio is $98/50 = 1.96$.

Its positive square root is 1.4.

Answer: Factor 1.4; $P = 50(1.4)^{t-1}$.

(b) Predict P at 4 and calculate the residual. [2 marks]

$50(1.4)^3 = 137.2$.

$142 - 137.2 = 4.8$.

Answer: Prediction 137.2; residual 4.8.

(c) Why is the first observation not automatically the value at t = 0? [2 marks]

The time origin matters; the model uses exponent $t - 1$.

Answer: It was measured at t = 1.

10. A club fitted $N = 200 + 15t$ to weekly attendance for t from 0 to 8. Its room holds 400 people. [6 marks]

(a) Predict attendance at t = 20 using the formula. [2 marks]

$200 + 15(20) = 500$.

Answer: 500.

(b) Explain two reasons to question this prediction. [2 marks]

20 is outside the data range.

500 exceeds the stated 400-person bound.

Answer: It extrapolates beyond 0-8 weeks and exceeds room capacity.

(c) Find when the rule reaches capacity. [2 marks]

$200 + 15t = 400$ gives $t = 200/15$.

Answer: About 13.33 weeks.