Counting paths with restrictions

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Edges are A→B, A→C, B→D and C→D. List two-edge walks from A to D. [2 marks]

B and C are the two possible intermediate vertices.

Each gives one permitted walk.

Answer: A→B→D and A→C→D.

2. For $A=\begin{pmatrix}0&1&1\\0&0&1\\1&0&0\end{pmatrix}$, find $(A^2)_{AA}$. [2 marks]

Row A times column A gives $0(0)+1(0)+1(1)=1$.

The walk is A→C→A.

Answer: 1.

3. For $A=\begin{pmatrix}0&1&1\\0&0&1\\1&0&0\end{pmatrix}$, find $(A^2)_{BC}$. [2 marks]

Row B times column C gives $0(1)+0(1)+1(0)=0$.

No such two-edge walk exists.

Answer: 0.

4. A route must not revisit its start. Why might diagonal entries of A^2 be excluded from a total? [2 marks]

A diagonal entry has identical start and finish.

That violates the no-revisit condition.

Answer: They represent walks returning to their starting vertex.

5. There are two distinct ferry services A→B and three B→C services. Count A→B→C service combinations. [2 marks]

There are 2 first-service choices.

Each has 3 second-service choices: $2(3) = 6$.

Answer: 6.

6. Edges are A→B, A→C, B→C, B→D and C→D. List all A-to-D paths with no repeated vertices. [2 marks]

First branch to B or C.

From B choose D or go via C; from C go to D.

Answer: A→B→D; A→C→D; A→B→C→D.