Minimum spanning trees including Prim's algorithm

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Find a minimum spanning tree for A-B (3), A-C (4), B-C (2). [2 marks]

The two cheapest edges connect all vertices.

They do not create a cycle.

Answer: A-B and B-C, total 5.

2. Can all three sides of a triangle form a tree on its three vertices? [2 marks]

They form a cycle.

A three-vertex tree has two edges.

Answer: No.

3. A connected selection has 5 vertices and 4 edges. What must be checked before calling it a spanning tree of the original network? [2 marks]

Spanning means none is omitted.

Connectedness and $n - 1$ edges then establish a tree.

Answer: That all original vertices are included.

4. Prim has reached A and B. Candidates are A-C (7), B-D (5), C-D (1). What is next? [2 marks]

C-D does not touch the current tree.

B-D is the cheapest valid joining edge.

Answer: B-D (5).

5. A minimum tree costs 120 dollars. Adding a backup link between two existing vertices costs 18 dollars. Give the new cost and explain whether the selection remains a tree. [2 marks]

$120 + 18 = 138$.

The extra link creates a cycle and an alternative route.

Answer: 138 dollars; it is no longer a tree.

6. Two spanning trees have the same minimum total 24 but different edges. Must one be wrong? [2 marks]

Tied weights can produce multiple optimum trees.

Check connectedness and the absence of cycles for each.

Answer: No.