Matrices and networks - Practice test 2

Name: _____ Date: _____

10 questions | 40 marks | 65 minutes

An original mixed chapter paper. Show calculations, state units and justify network decisions using the supplied information.

A scientific calculator may be used.

1. $A = \begin{pmatrix} 2 & x \\ 5 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 7 & 9 \\ 3 & 4 \end{pmatrix}$. [4 marks]

(a) Find x if the top-right entry of $A + B$ is 15. [2 marks]

(b) Find the remaining three entries of $A + B$. [2 marks]

2. Opening stock $S = \begin{pmatrix} 20 & 15 \\ 12 & 18 \end{pmatrix}$, deliveries $D = \begin{pmatrix} 4 & 8 \\ 6 & 3 \end{pmatrix}$ and sales $R = \begin{pmatrix} 9 & 7 \\ 5 & 11 \end{pmatrix}$. [4 marks]

(a) Find closing stock $S + D - R$. [2 marks]

(b) Use a total-stock check to verify the result. [2 marks]

3. Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$. [4 marks]

(a) State the product order and calculate AB. [2 marks]

(b) Does BA exist? Explain. [2 marks]

4. Rows in $Q = \begin{pmatrix} 3 & 4 \\ 5 & 2 \end{pmatrix}$ are orders; columns are coffee and tea boxes. Prices are 12 and 8 dollars per box. [4 marks]

(a) Calculate the two costs using a price column. [2 marks]

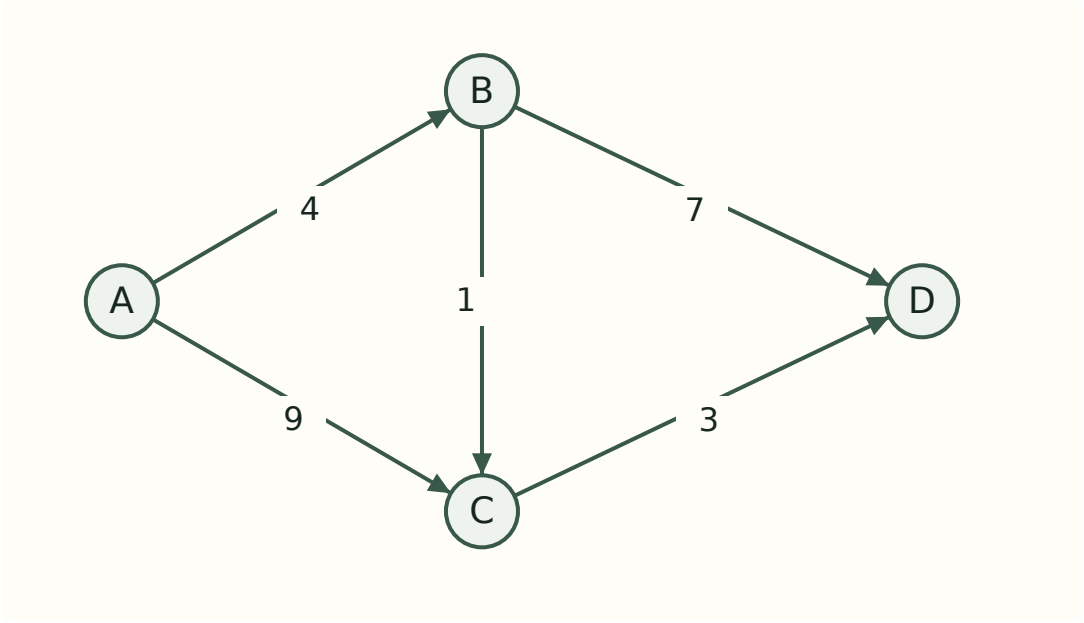
(b) Tea rises to 10 dollars per box. Find the increase in the combined cost. [2 marks]

5. A simple undirected graph has vertices A, B, C, D, E, F and edges AB, BC, CA, DE. [4 marks]

(a) State the components, including isolated vertices. [2 marks]

(b) What is the minimum number of new edges needed to connect the whole graph? Give an example. [2 marks]

6. The arrows show permitted bus connections; weights are minutes. [4 marks]



(a) Find the duration A→B→C→D and compare with A→C→D. [2 marks]

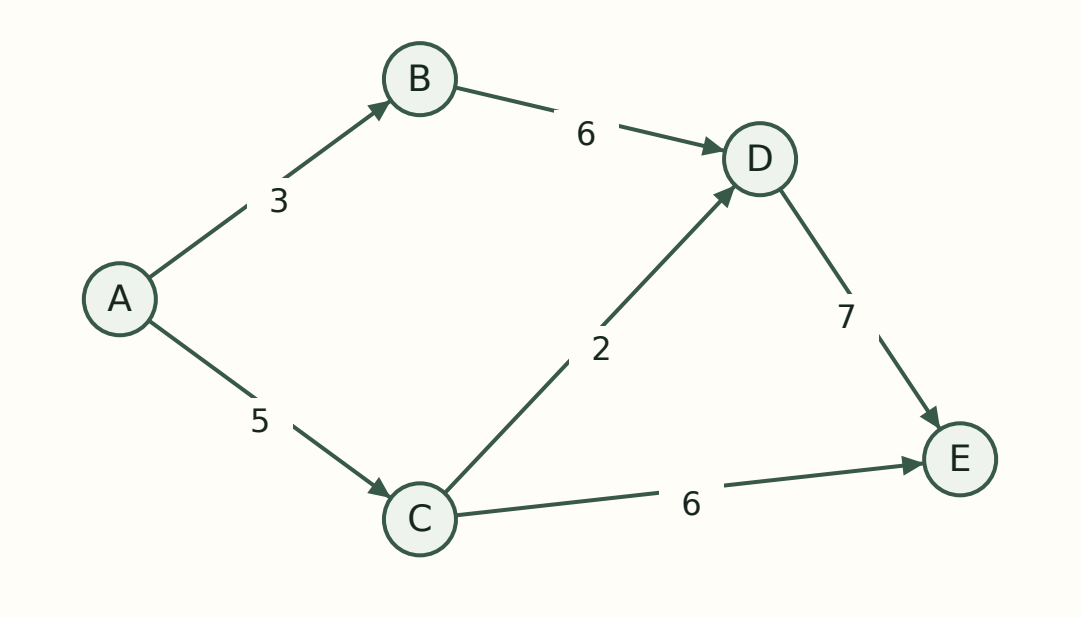
(b) If B→C is closed, compare the two remaining A-to-D paths. [2 marks]

7. Edges are A→B, A→C, B→A, B→D and C→D. No other edges exist. [4 marks]

(a) List all A-to-D paths with no repeated vertices. [2 marks]

(b) List an A-to-D four-edge walk that is not one of those paths. [2 marks]

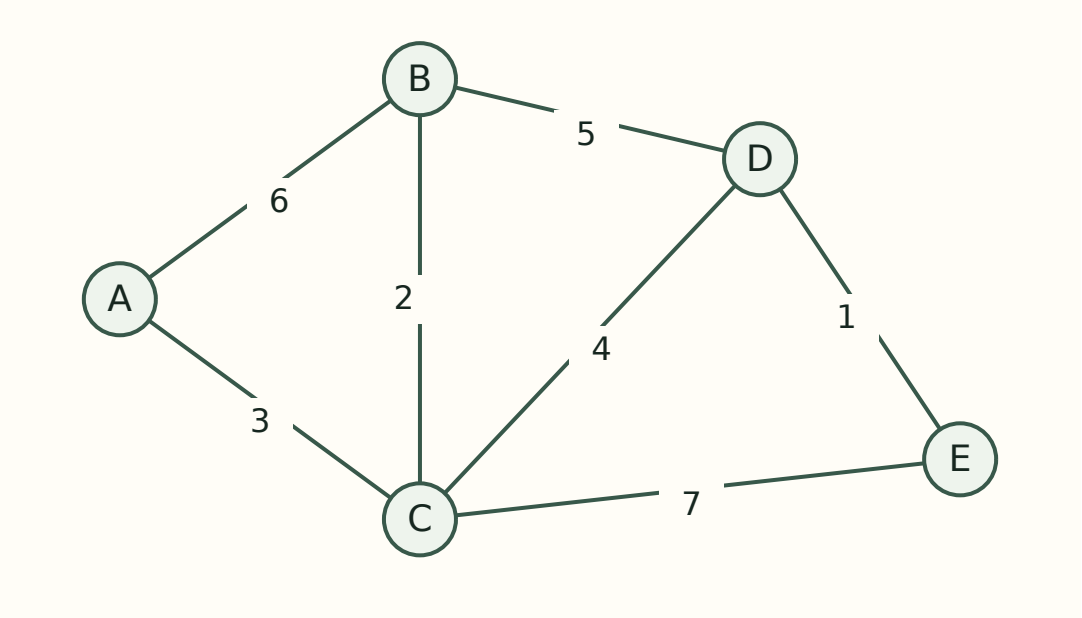
8. This directed network is acyclic. [4 marks]



(a) Find a longest A-to-E path. [2 marks]

(b) Why would allowing an extra E→A edge of positive weight make the unrestricted longest-walk question different? [2 marks]

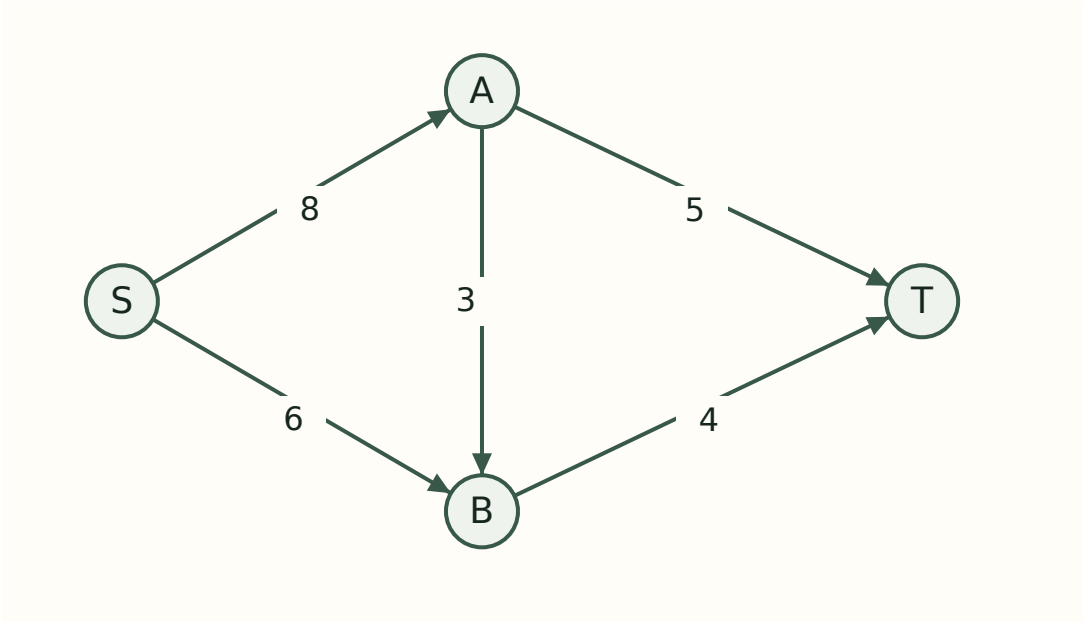
9. Connection weights are installation costs in thousands of dollars. [4 marks]



(a) Apply Prim from A and report a minimum tree. [2 marks]

(b) After reaching {A, B, C}, explain why D-E 1 cannot be chosen before C-D 4. [2 marks]

10. Directed pipe capacities are L/s. [4 marks]



(a) Show that maximum flow is 9 L/s. [2 marks]

(b) Only S→A is widened from 8 to 12. Does the maximum increase? Explain. [2 marks]