Matrices and networks - Practice test 3

10 questions | 40 marks | 65 minutes

An original mixed chapter paper. Show calculations, state units and justify network decisions using the supplied information.

Equivalent correct methods earn the corresponding marks.

1. A library records books borrowed. Rows are fiction/non-fiction and columns are Monday/Tuesday/Wednesday. The entries are $\begin{pmatrix} 18 & 22 & 16 \\ 9 & 14 & 11 \end{pmatrix}$. [4 marks]

(a) Interpret the entry in row 2, column 3 and find Wednesday's total. [2 marks]

Locate row 2, column 3.

Add Wednesday entries $16 + 11 = 27$.

Answer: 11 non-fiction books; Wednesday total 27.

(b) What extra information is needed to turn the borrowing matrix into a matrix of money earned? [2 marks]

Counts alone are not revenue.

Prices and charging conditions are needed before calculating dollars.

Answer: A valid charging rule, such as a charge per loan and any exceptions.

2. A clinic has opening medical packs $S = \begin{pmatrix} 15 & 20 \\ 12 & 18 \end{pmatrix}$ and receives $D = \begin{pmatrix} 6 & 4 \\ 8 & 3 \end{pmatrix}$. Usage is $U = \begin{pmatrix} 9 & 11 \\ 7 & 10 \end{pmatrix}$. [4 marks]

(a) Calculate the final stock matrix. [2 marks]

Use $S + D - U$.

Calculate each of the four categories separately.

Answer: $\begin{pmatrix} 12 & 13 \\ 13 & 11 \end{pmatrix}$.

(b) Each remaining pack costs 4 dollars. Find the total value. [2 marks]

Remaining packs total $12 + 13 + 13 + 11 = 49$.

$49(4) = 196$.

Answer: 196 dollars.

3. A product satisfies $\begin{pmatrix} x & 1 \\ 2 & y \end{pmatrix}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 19 \\ 18 \end{pmatrix}$. [4 marks]

(a) Find x and y. [2 marks]

First row: $3x + 4 = 19$.

Second row: $6 + 4y = 18$.

Answer: $x = 5$, $y = 3$.

(b) Substitute your values to check both rows. [2 marks]

$3(5) + 4 = 19$.

$6 + 4(3) = 18$.

Answer: Both give the required 19 and 18.

4. Order rows in $Q = \begin{pmatrix} 2 & 1 & 3 \\ 4 & 0 & 2 \end{pmatrix}$ represent orders of three item types. Their prices are $\begin{pmatrix} 5 \\ 8 \\ 3 \end{pmatrix}$ dollars. [4 marks]

(a) Calculate the order costs. [2 marks]

$2(5) + 1(8) + 3(3) = 27$.

$4(5) + 0(8) + 2(3) = 26$.

Answer: 27 and 26 dollars.

(b) Both orders are paid from a 60-dollar budget with one 5-dollar total delivery fee. Find the balance. [2 marks]

Subtotal $27 + 26 = 53$.

Total $53 + 5 = 58$; balance $60 - 58 = 2$.

Answer: 2 dollars.

5. A student claims that a simple undirected four-vertex graph has degrees 3, 3, 3 and 2. [4 marks]

(a) Explain why the record is impossible. [2 marks]

Each edge contributes two ends.

An undirected graph must have an even degree sum.

Answer: The degree sum 11 is odd.

(b) Change only the last degree to make a possible simple graph, and describe one. [2 marks]

The complete four-vertex graph has degree 3 at every vertex.

It has 6 edges and degree sum 12.

Answer: Change 2 to 3; connect every pair of the four vertices.

6. A council compares two A-to-D routes. Route A-B-D has distances 2 km and 3 km but installation costs 9,000 and 8,000 dollars. Route A-C-D has distances 4 km and 4 km but costs 5,000 and 6,000 dollars. [4 marks]

(a) Find the shortest-distance route and total. [2 marks]

Its distance is $2 + 3 = 5$.

The other route totals 8 km.

Answer: A-B-D;5 km.

(b) Find the lowest-installation-cost route and explain the difference. [2 marks]

Costs are 17,000 versus 11,000 dollars.

The two objectives use different edge weights.

Answer: A-C-D;11,000 dollars.

7. There are directed edges A→B, A→C, B→C, B→D, C→D and D→E. [4 marks]

(a) List all A-to-E paths with no repeated vertices. [2 marks]

Branch first to B or C.

Every path must finish with D→E.

Answer: A→B→D→E; A→C→D→E; A→B→C→D→E.

(b) How many of them use exactly three edges, and how many use four? [2 marks]

Count the connections in each listed path.

The via-B-and-C path is the four-edge case.

Answer: Two use 3 edges; one uses 4.

8. Directed travel times are S→A 6, S→B 2, B→A 1, A→T 3 and B→T 7 minutes. [4 marks]

(a) Find a shortest route and its time. [2 marks]

Routes take 9, 9 and 6 minutes.

Select the total $2 + 1 + 3 = 6$.

Answer: S→B→A→T;6 min.

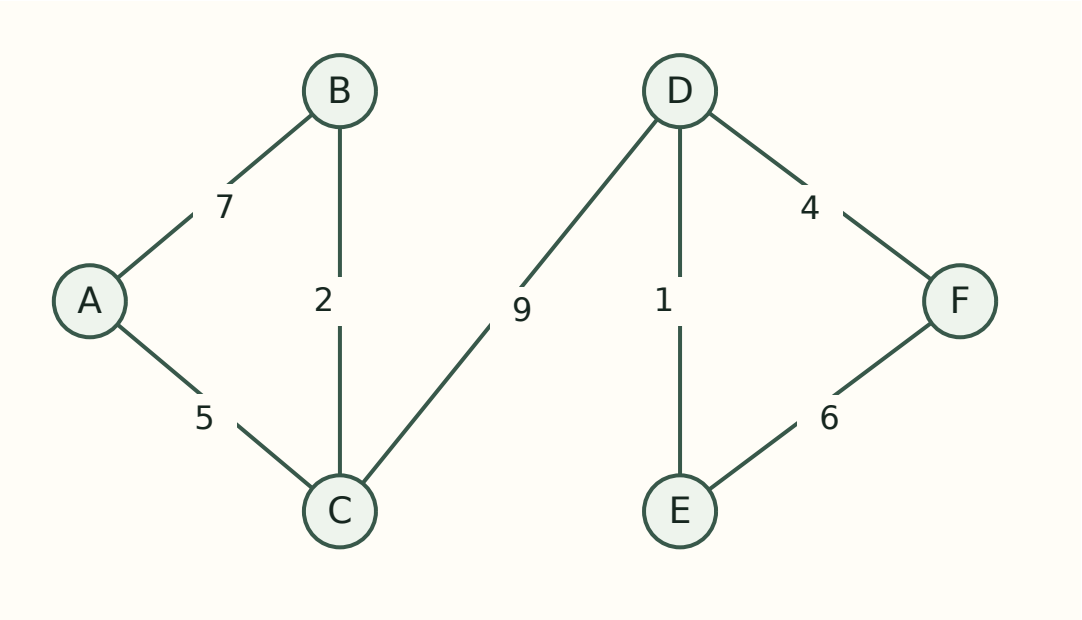
(b) If the B→A time rises to 5 minutes, reassess the decision. [2 marks]

The via-B-and-A route becomes $2 + 5 + 3 = 10$.

The other routes remain 9.

Answer: S→A→T or S→B→T, each 9 min.

9. Weights are installation costs. Use the undirected network shown. [4 marks]



(a) Explain why every spanning tree must include C-D. [2 marks]

Removing it separates {A, B, C} from {D, E, F}.

A spanning tree must connect those groups.

Answer: C-D is the only link between the two groups.

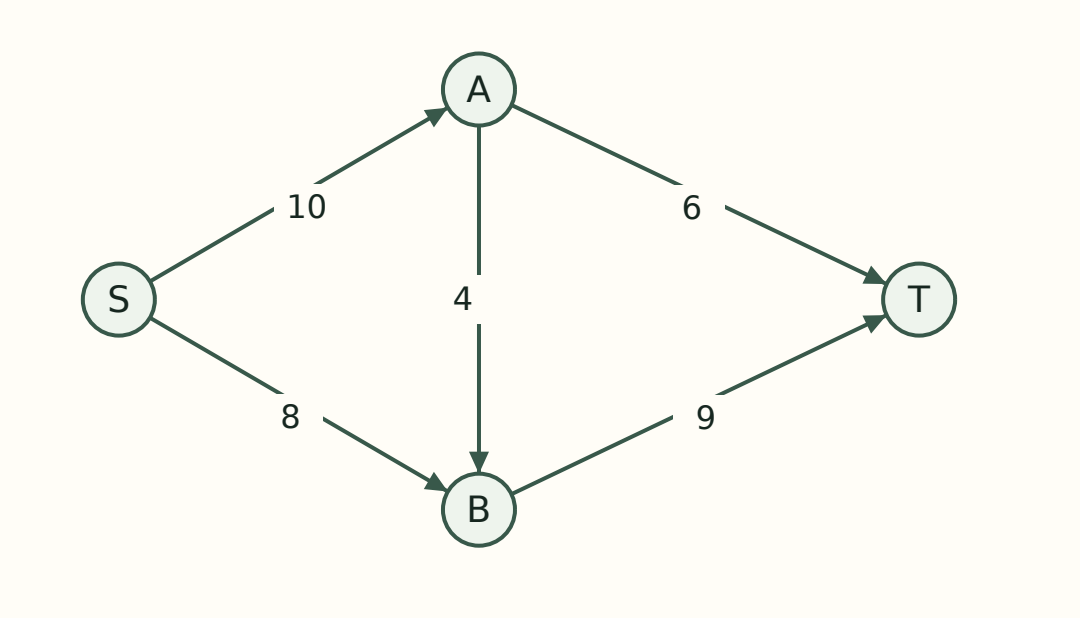
(b) Find a minimum spanning tree and its total. [2 marks]

Use the cheapest two edges connecting each triangle.

The forced bridge costs 9; total $5 + 2 + 9 + 1 + 4 = 21$.

Answer: A-C 5, B-C 2, C-D 9, D-E 1, D-F 4; total 21.

10. Capacities are units/min. A proposed assignment is SA 10, SB 8, AB 4, AT 6, BT 9. [4 marks]



(a) Explain why that assignment is infeasible even though no stated edge flow exceeds its capacity. [2 marks]

At B, inflow is $8 + 4 = 12$.

The difference 3 has no permitted storage or exit.

Answer: B receives 12 but sends 9; conservation fails.

(b) Correct it to a maximum flow and prove the value. [2 marks]

Both intermediate vertices conserve flow.

The sink cut has capacity $6 + 9 = 15$.

Answer: Use SA 10, SB 5, AB 4, AT 6, BT 9; maximum 15.