Matrices and networks - Practice test 3

Name: _____ Date: _____

10 questions | 40 marks | 65 minutes

An original mixed chapter paper. Show calculations, state units and justify network decisions using the supplied information.

A scientific calculator may be used.

1. A library records books borrowed. Rows are fiction/non-fiction and columns are Monday/Tuesday/Wednesday. The entries are $\begin{pmatrix} 18 & 22 & 16 \\ 9 & 14 & 11 \end{pmatrix}$. [4 marks]

(a) Interpret the entry in row 2, column 3 and find Wednesday's total. [2 marks]

(b) What extra information is needed to turn the borrowing matrix into a matrix of money earned? [2 marks]

2. A clinic has opening medical packs $S = \begin{pmatrix} 15 & 20 \\ 12 & 18 \end{pmatrix}$ and receives $D = \begin{pmatrix} 6 & 4 \\ 8 & 3 \end{pmatrix}$. Usage is $U = \begin{pmatrix} 9 & 11 \\ 7 & 10 \end{pmatrix}$. [4 marks]

(a) Calculate the final stock matrix. [2 marks]

(b) Each remaining pack costs 4 dollars. Find the total value. [2 marks]

3. A product satisfies $\begin{pmatrix} x & 1 \\ 2 & y \end{pmatrix}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 19 \\ 18 \end{pmatrix}$. [4 marks]

(a) Find x and y. [2 marks]

(b) Substitute your values to check both rows. [2 marks]

4. Order rows in $Q = \begin{pmatrix} 2 & 1 & 3 \\ 4 & 0 & 2 \end{pmatrix}$ represent orders of three item types. Their prices are $\begin{pmatrix} 5 \\ 8 \\ 3 \end{pmatrix}$ dollars. [4 marks]

(a) Calculate the order costs. [2 marks]

(b) Both orders are paid from a 60-dollar budget with one 5-dollar total delivery fee. Find the balance. [2 marks]

5. A student claims that a simple undirected four-vertex graph has degrees 3, 3, 3 and 2. [4 marks]

(a) Explain why the record is impossible. [2 marks]

(b) Change only the last degree to make a possible simple graph, and describe one. [2 marks]

6. A council compares two A-to-D routes. Route A-B-D has distances 2 km and 3 km but installation costs 9,000 and 8,000 dollars. Route A-C-D has distances 4 km and 4 km but costs 5,000 and 6,000 dollars. [4 marks]

(a) Find the shortest-distance route and total. [2 marks]

(b) Find the lowest-installation-cost route and explain the difference. [2 marks]

7. There are directed edges A→B, A→C, B→C, B→D, C→D and D→E. [4 marks]

(a) List all A-to-E paths with no repeated vertices. [2 marks]

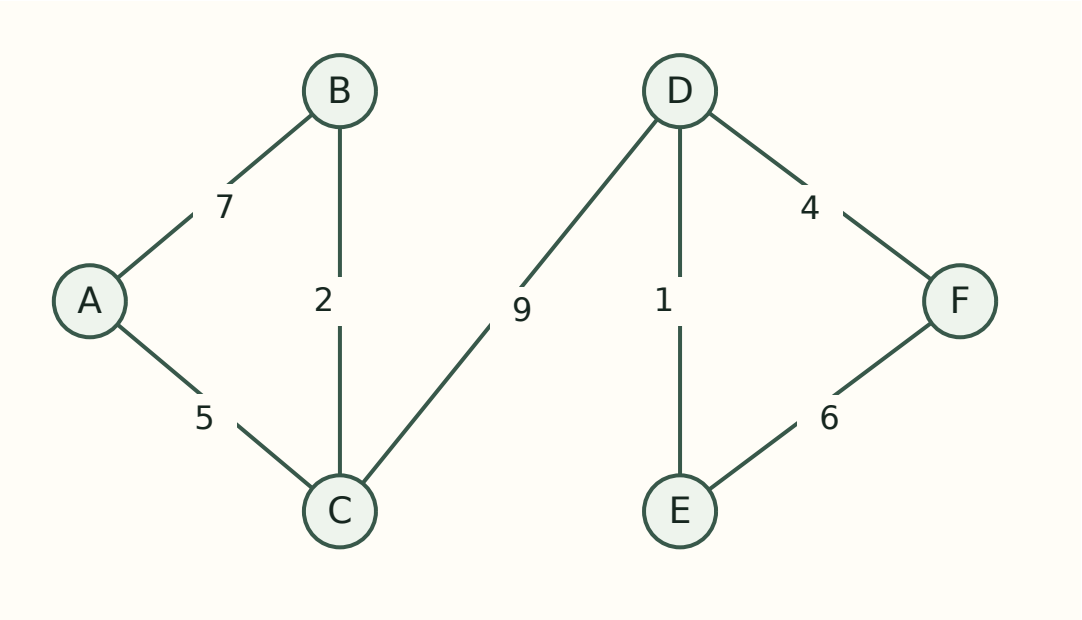
(b) How many of them use exactly three edges, and how many use four? [2 marks]

8. Directed travel times are S→A 6, S→B 2, B→A 1, A→T 3 and B→T 7 minutes. [4 marks]

(a) Find a shortest route and its time. [2 marks]

(b) If the B→A time rises to 5 minutes, reassess the decision. [2 marks]

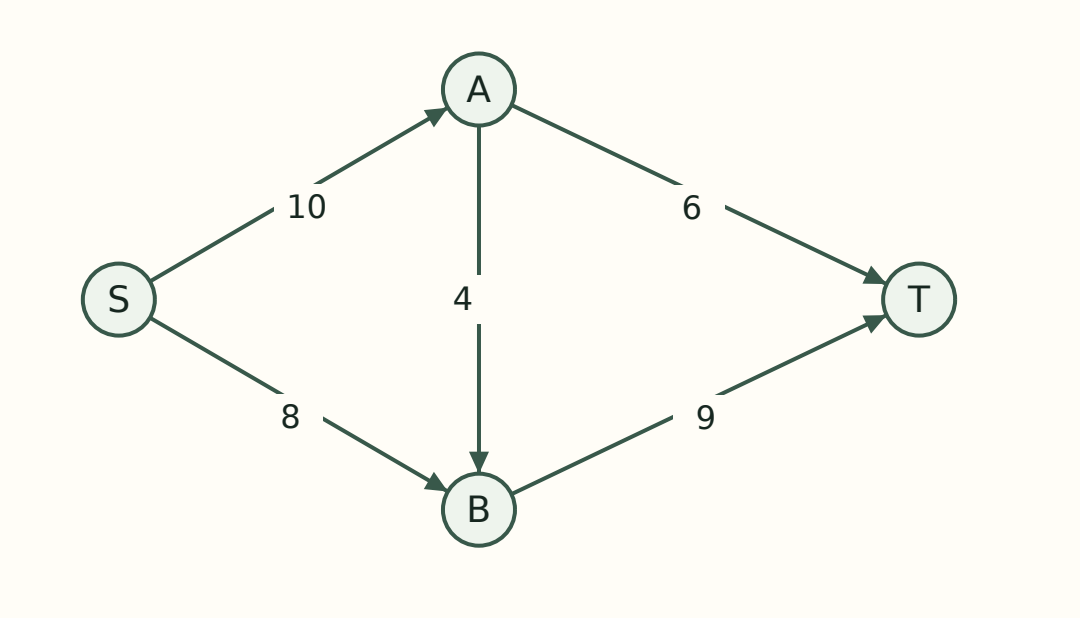
9. Weights are installation costs. Use the undirected network shown. [4 marks]



(a) Explain why every spanning tree must include C-D. [2 marks]

(b) Find a minimum spanning tree and its total. [2 marks]

10. Capacities are units/min. A proposed assignment is SA 10, SB 8, AB 4, AT 6, BT 9. [4 marks]



(a) Explain why that assignment is infeasible even though no stated edge flow exceeds its capacity. [2 marks]

(b) Correct it to a maximum flow and prove the value. [2 marks]