Matrices and networks - Practice test 4

Name: _____ Date: _____

10 questions | 40 marks | 65 minutes

An original mixed chapter paper. Show calculations, state units and justify network decisions using the supplied information.

A scientific calculator may be used.

1. $A = \begin{pmatrix} 3 & 1 & 0 \\ 2 & 5 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 0 & 3 \\ 4 & 1 \end{pmatrix}$. [4 marks]

(a) State the orders of AB and BA. [2 marks]

(b) Calculate $(AB)_{21}$ and $(BA)_{32}$. [2 marks]

2. An order row is $Q = \begin{pmatrix} 2 & 5 \end{pmatrix}$ for bread and milk. Bread costs 4 dollars and milk 3 dollars. A student uses price column $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$. [4 marks]

(a) State the student’s result and the correct result. [2 marks]

(b) Explain the modelling error. [2 marks]

3. Let $A = \begin{pmatrix} 1 & 3 \\ 2 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ 0 & 4 \end{pmatrix}$. [4 marks]

(a) Calculate AB. [2 marks]

(b) Calculate BA and give a numerical counterexample to commutativity. [2 marks]

4. Rows represent stores 1 and 2; columns represent products A and B. Opening stock $S = \begin{pmatrix} 12 & 8 \\ 5 & 14 \end{pmatrix}$ is followed by two identical deliveries $D = \begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix}$ and sales $R = \begin{pmatrix} 8 & 7 \\ 6 & 9 \end{pmatrix}$. [4 marks]

(a) Write and calculate the closing-stock expression. [2 marks]

(b) If the first product costs 5 dollars and the second 2 dollars, find each store’s closing value. [2 marks]

5. An undirected graph has edges AB, BC, CD, DA and AC. [4 marks]

(a) Is any edge a bridge? Explain. [2 marks]

(b) If AC and AB are both removed, is the remaining graph connected? [2 marks]

6. Travel times are A→B 3 min, B→A 8 min, B→C 5 min, C→B 4 min, A→C 10 min and C→A 7 min. [4 marks]

(a) Find the fastest A-to-C journey. [2 marks]

(b) Find the fastest C-to-A journey and explain why it need not reverse the first. [2 marks]

7. Rows and columns of $N = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ are A, B, C, with rows as starting vertices. [4 marks]

(a) Find $(N^3)_{CB}$. [2 marks]

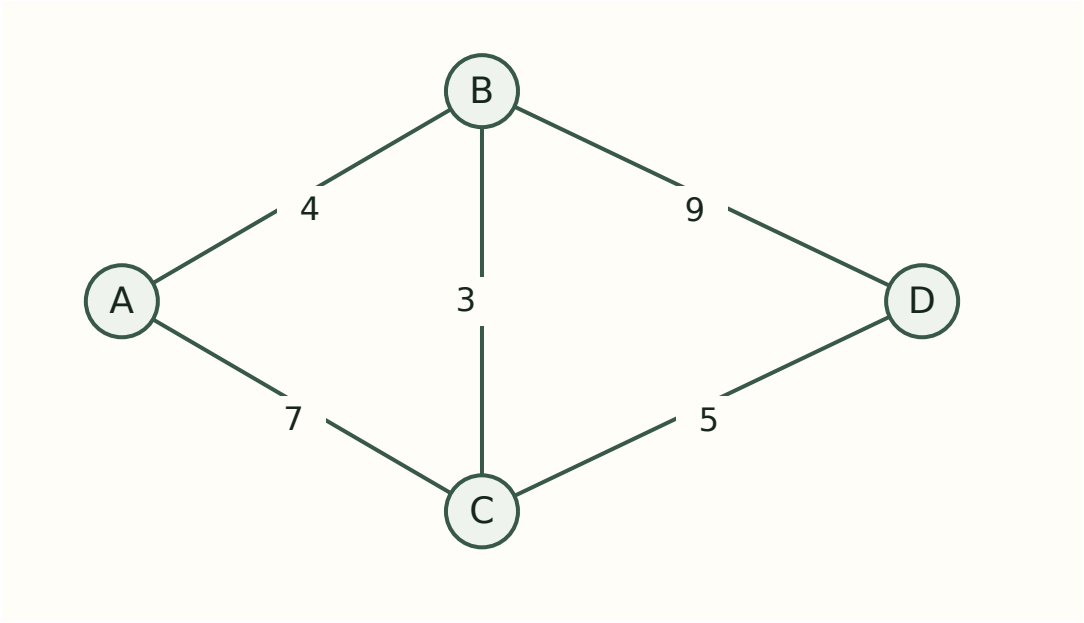
(b) Find $(N^3)_{CC}$ and list the corresponding walk. [2 marks]

8. A directed acyclic graph has A→B 2, A→C 6, B→C 1, B→D 8, C→D 3, D→E 2 and C→E 10. [4 marks]

(a) Find a shortest A-to-E path. [2 marks]

(b) Find a longest A-to-E path. [2 marks]

9. Connection costs are shown. [4 marks]



(a) Find a minimum spanning tree. [2 marks]

(b) A new A-D edge costs 2. Find the new minimum tree. [2 marks]

10. A directed network has S→A 7, S→B 5, A→B 3, A→T 4 and B→T 8. Capacities are units/hour. [4 marks]

(a) Find the capacity of the cut with source side {S, A} and sink side {B, T}. [2 marks]

(b) Show a feasible flow reaching 12 and hence prove the maximum. [2 marks]