Linear models and optimisation - Practice test 1

10 questions | 48 marks | 70 minutes

A balanced paper building from equations and comparisons to constrained decisions.

Equivalent correct methods earn the corresponding marks.

1. A display company charges AUD 65 for setup and AUD 18 per stand. [3 marks]

(a) Define variables and write a linear cost rule. [2 marks]

Let C be dollars and n the number of stands.

65 is fixed; each stand adds 18.

Answer: $C = 65 + 18n$ for whole-number $n \geq 0$.

(b) Find the cost of 9 stands. [1 mark]

$65 + 18(9) = 227$.

Answer: AUD 227.

2. Solve the simultaneous equations $2x + 3y = 24$ and $x + y = 9$. [4 marks]

(a) Find x and y. [3 marks]

Substitute $x = 9 - y$: $18 - 2y + 3y = 24$.

Then y=6 and x=3.

Answer: $(3, 6)$.

(b) Verify both equations. [1 mark]

$2(3) + 3(6) = 24$ and $3 + 6 = 9$.

Answer: Both hold.

3. Plan A costs $A = 48 + 2n$ and plan B costs $B = 18 + 5n$, in dollars. [4 marks]

(a) Find their equal-cost point. [2 marks]

$48 + 2n = 18 + 5n$ gives n=10; substitute.

Answer: 10 uses; AUD 68.

(b) Which plan is cheaper for 15 uses, and by how much? [2 marks]

$A = 78$ and $B = 93$.

Answer: A by AUD 15.

4. A maker can spend at most 42 hours. A small unit uses 2 hours and a large unit 5 hours. [3 marks]

(a) Write the time restriction for x small and y large units. [2 marks]

Add the hours used by each product.

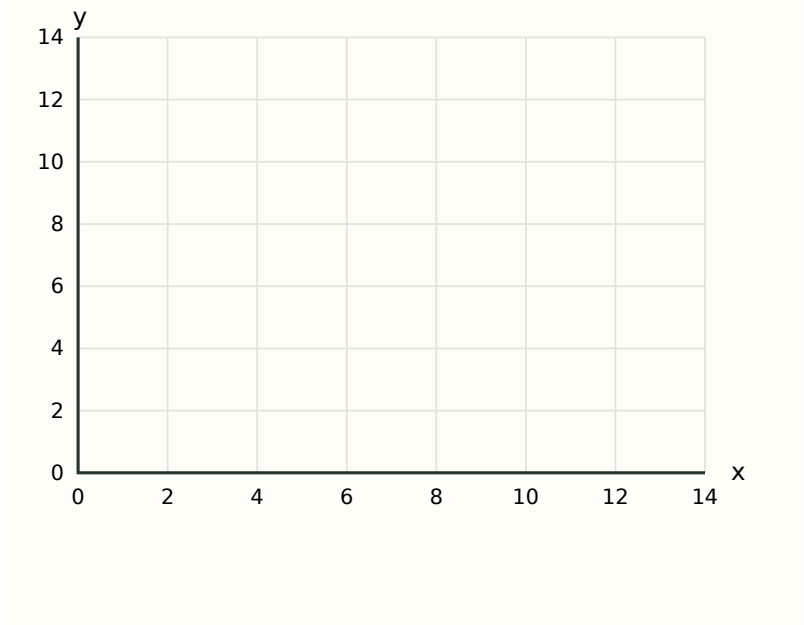
Answer: $2x + 5y \le 42$, with non-negative integer x,y.

(b) Does $(6, 6)$ meet this restriction? [1 mark]

$2(6) + 5(6) = 42$.

Answer: Yes, exactly.

5. Use $x, y \geq 0$, $x + y \leq 12$ and $x \leq 8$. [4 marks]



(a) Sketch the feasible region and list all its vertices. [3 marks]

Draw x=8 and x+y=12; retain the first-quadrant overlap.

Answer: $(0, 0), (8, 0), (8, 4), (0, 12)$.

(b) Explain why $(10, 1)$ is excluded even though its sum is below 12. [1 mark]

Every constraint must hold.

Answer: It violates x at most 8.

6. A shop makes AUD 14 profit on A and AUD 9 on B. The feasible vertices are $(0, 0), (6, 0), (4, 5), (0, 7)$. [4 marks]

(a) Write and evaluate the objective at the vertices. [3 marks]

Use profit coefficients, not quantities alone.

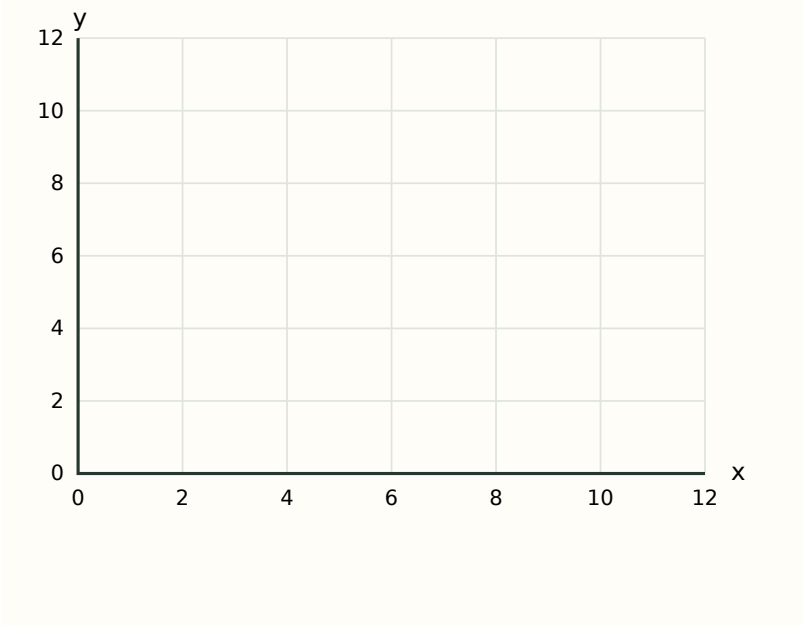
Answer: $P = 14x + 9y$; values 0,84,101,63.

(b) State the best decision and profit. [1 mark]

Choose the largest objective value.

Answer: 4 of A and 5 of B; AUD 101.

7. A workshop makes x small boxes and y large boxes. Each uses one sheet; a small box needs 2 assembly units and a large one 1. There are 10 sheets and 14 assembly units. Unit profits are AUD 5 and AUD 3. [8 marks]



(a) State the model, including variable restrictions. [3 marks]

Define x and y as described.

Translate each separate resource or minimum requirement.

Answer: $x + y \leq 10$, $2x + y \leq 14$, $x, y \geq 0$; Maximise $P = 5x + 3y$. Counts x and y must be integers.

(b) Find and justify the optimal decision. Give the objective value. [5 marks]

Feasible vertices: (0, 0), (0, 10), (4, 6), (7, 0).

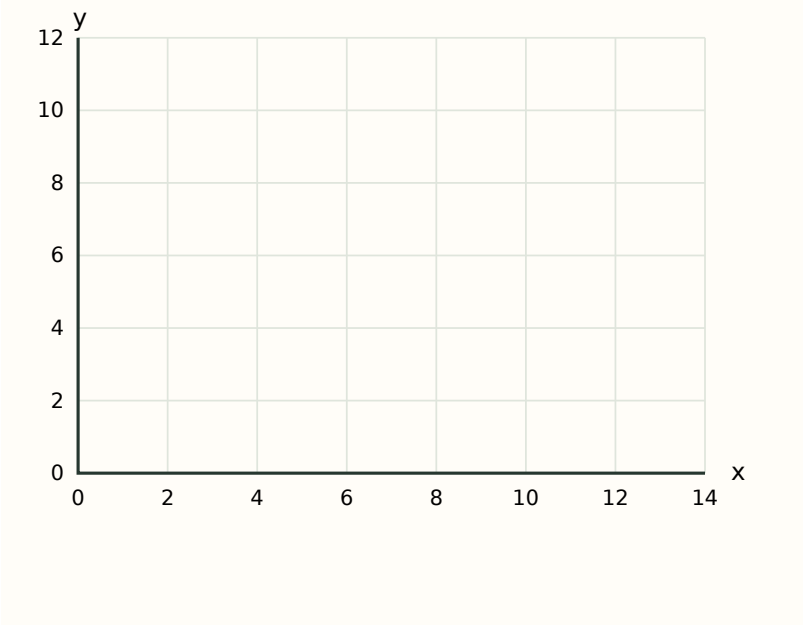
(0, 0) gives 0; (0, 10) gives 30; (4, 6) gives 38; (7, 0) gives 35

The largest vertex value is 38.

The continuous optimum has whole-number coordinates, so it also gives the optimum for the item counts.

Answer: (4, 6); objective 38.

8. A blend uses x litres of A and y litres of B. It must total at least 8 litres and supply at least 12 nutrient units; A has 1 unit/litre and B has 2. Costs are AUD 3 and AUD 4 per litre. [8 marks]



(a) State the model, including variable restrictions. [3 marks]

Define x and y as described.

Translate each separate resource or minimum requirement.

Answer: $x + y \geq 8$, $x + 2y \geq 12$, $x, y \geq 0$; Minimise $C = 3x + 4y$.

(b) Find and justify the optimal decision. Give the objective value. [5 marks]

Feasible vertices: (0, 8), (4, 4), (12, 0).

(0, 8) gives 32; (4, 4) gives 28; (12, 0) gives 36

The smallest vertex value is 28.

Along each unbounded horizontal or vertical boundary ray the positive cost increases. The minimum is found among the lower-bound corners.

Answer: (4, 4); objective 28.

9. For integer x,y, maximise $P = 3x + 4y$ subject to $2x + y \leq 7$, $x + 2y \leq 7$ and non-negativity. [6 marks]

(a) Show why rounding the continuous corner up to $(3, 3)$ fails. [2 marks]

Each left side is 9, greater than 7.

Answer: Both resource bounds fail.

(b) Find the integer optimum using a table. [4 marks]

For x=0,1,2,3, largest feasible y=3,3,2,1.

Objective values are 12,15,14,13.

Answer: $(1, 3)$ with P=15.

10. A linear model recommends producing 60 gift packs, assuming all packs sell and each uses the same labour time. [4 marks]

(a) State two assumptions that should be checked in practice. [2 marks]

The sales and labour assumptions directly affect profit and feasibility.

Answer: Demand for at least 60 packs; constant labour use per pack.

(b) Orders cap gift-pack demand at 45. Explain the required model change. [2 marks]

The previous recommendation may no longer be feasible.

Answer: Add the supported bound of 45 packs and re-optimise.