Linear models and optimisation - Practice test 2

10 questions | 47 marks | 70 minutes

Graphs, minimum requirements and the meaning of linear comparisons.

Equivalent correct methods earn the corresponding marks.

1. A water reservoir contains 720 litres and loses 24 litres per minute at a constant rate. [5 marks]

(a) Write volume V after t minutes and state the useful domain. [3 marks]

The loss is a negative gradient.

The tank empties at 720/24=30.

Answer: $V = 720 - 24t,\ 0 \le t \le 30$.

(b) When does the volume reach 240 litres? [2 marks]

$720 - 24t = 240$ gives $24t = 480$.

Answer: 20 minutes.

2. A graphing tool displays an intersection near $(2.5, 4.5)$ for $3x + y = 12$ and $x - y = -2$. [4 marks]

(a) Find the exact intersection algebraically. [3 marks]

Add to get $4x = 10$; then use x-y=-2.

Answer: $(5/2, 9/2)$.

(b) Explain why graph coordinates should be checked. [1 mark]

Substitution verifies both original conditions.

Answer: Plot resolution and rounding can hide the exact intersection.

3. Two hire plans cost $A = 80 + 6d$ and $B = 35 + 9d$, with d in days. [4 marks]

(a) Find the equal-cost duration and cost. [2 marks]

$80 + 6d = 35 + 9d$ gives d=15.

Answer: 15 days; AUD 170.

(b) Choose the cheaper plan for 8 days. [2 marks]

Evaluate both rules at d=8.

Answer: B, costing AUD 107 rather than AUD 128.

4. Solve and interpret $36 - 3x < 15$. [3 marks]

(a) Find the solution interval. [2 marks]

$-3x < -21$; divide by -3 and reverse.

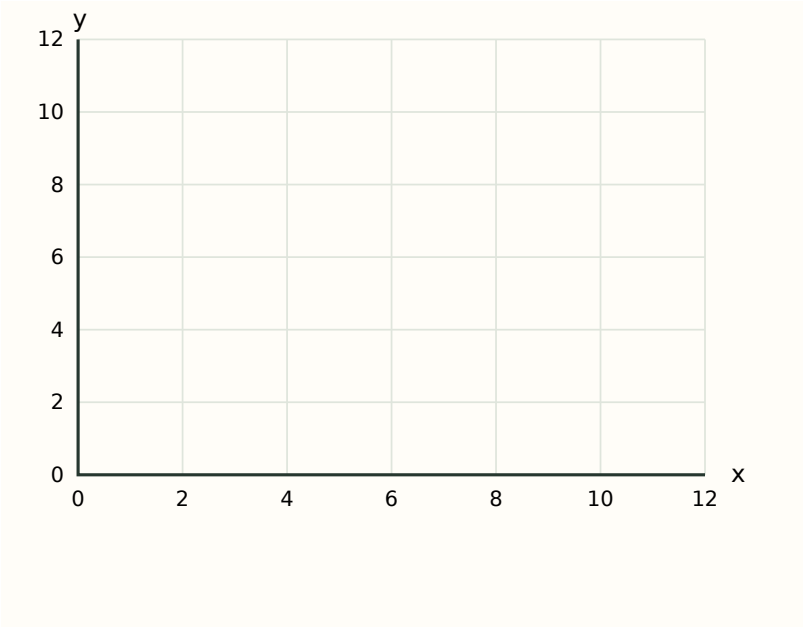
Answer: $x > 7$.

(b) If x counts whole boxes, find the least permitted x. [1 mark]

The strict boundary excludes 7.

Answer: 8.

5. The feasible set is $x, y \geq 0$, $x + y \geq 6$, $y \leq 5$. [4 marks]



(a) List its finite corner points and explain why the set is unbounded. [3 marks]

Intersect x+y=6 with y=0 and y=5.

Answer: Corners $(6, 0), (1, 5)$; x can increase without bound.

(b) Does unboundedness itself prove that no minimum cost exists? [1 mark]

A positive objective can still have a minimum on the lower boundary.

Answer: No.

6. On a feasible edge, $x + y = 10$ with x,y non-negative. The objective is $P = 6x + 6y$. [3 marks]

(a) Find P anywhere on the edge. [1 mark]

$P = 6(x + y) = 60$.

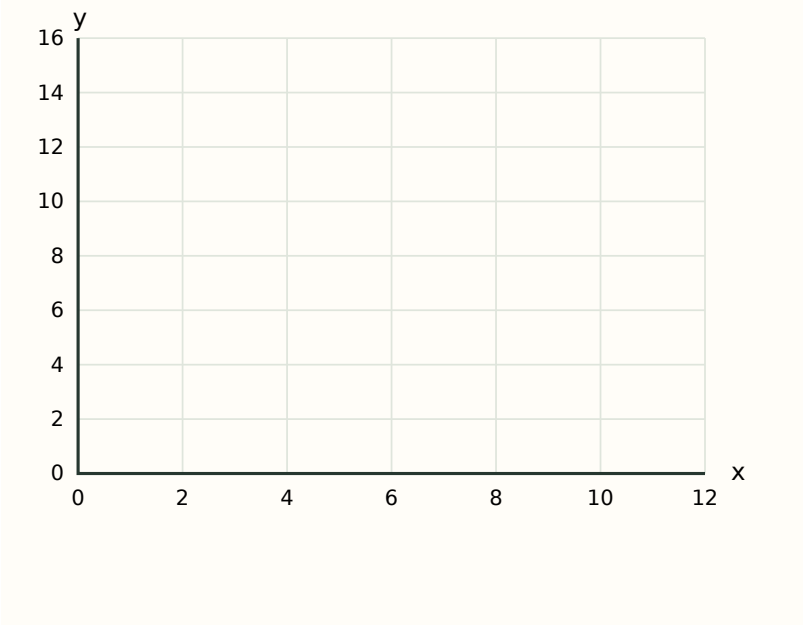
Answer: 60.

(b) If this is the greatest feasible value, describe the full set of optima. [2 marks]

Equal coefficients make the objective parallel to that edge.

Answer: Every point on the edge is optimal.

7. A farm mixes x tonnes of feed A and y tonnes of feed B. At least 10 tonnes are needed. A supplies 2 nutrient units/tonne and B 1; at least 14 units are required. A costs AUD 4 per tonne and B costs AUD 3 per tonne. [8 marks]



(a) State the model, including variable restrictions. [3 marks]

Define x and y as described.

Translate each separate resource or minimum requirement.

Answer: $x + y \geq 10$, $2x + y \geq 14$, $x, y \geq 0$; Minimise $C = 4x + 3y$.

(b) Find and justify the optimal decision. Give the objective value. [5 marks]

Feasible vertices: (0, 14), (4, 6), (10, 0).

(0, 14) gives 42; (4, 6) gives 34; (10, 0) gives 40

The smallest vertex value is 34.

Along each unbounded horizontal or vertical boundary ray the positive cost increases. The minimum is found among the lower-bound corners.

Answer: (4, 6); objective 34.

8. A manufacturer has constraints $x + y \leq 14$ and $3x + y \leq 24$, with x,y non-negative. Profit is $P = 7x + 4y$. [6 marks]

(a) Find the feasible vertices. [3 marks]

Subtract sloping equations to get 2 x=10, so (5,9).

Answer: $(0, 0), (8, 0), (5, 9), (0, 14)$.

(b) Find maximum profit and identify unused resources there. [3 marks]

Values are 0,56,71,56.

Resource use is 14 and 24.

Answer: Maximum 71 at (5,9); both bounds are active.

9. A class needs at least 58 seats. A small vehicle provides 10 seats for AUD 90 and a large vehicle 18 seats for AUD 140. At most 5 vehicles may be hired. [7 marks]

(a) Write an integer model minimising cost. [3 marks]

Translate seats and vehicle count separately.

Answer: Minimise $C = 90x + 140y$ with $10x + 18y \geq 58$, $x + y \leq 5$, non-negative integers.

(b) Find the least-cost plan. [4 marks]

For y=0 no plan fits; y=1 requires x=4 (500); y=2 requires x=3 (550); y=3 requires x=1 (510); y=4 costs 560 and y=5 costs 700.

Compare all feasible lower-cost candidates.

Answer: 4 small and 1 large; AUD 500.

10. A solution at $(5, 9)$ uses all 24 available assembly hours. Next week only 22 hours are available. [3 marks]

(a) Explain why the current solution cannot simply be reused. [1 mark]

Feasibility depends on the available hours.

Answer: It exceeds the new resource bound.

(b) Describe how to update the model and report. [2 marks]

Keep the other data unchanged unless evidence supports further changes.

Answer: Replace the 24-hour bound by 22, re-solve and compare the new objective and decision.