Linear models and optimisation - Practice test 3

10 questions | 49 marks | 70 minutes

Whole-number planning, competing limits and justified decisions.

Equivalent correct methods earn the corresponding marks.

1. The total cost of 4 display boards is AUD 104; the total for 9 boards is AUD 189. Assume a linear rule. [4 marks]

(a) Find the variable cost and fixed cost. [3 marks]

The gradient is $(189-104)/(9-4)=17$.

Use $104=4(17)+b$ to get b=36.

Answer: AUD 17 per board; AUD 36 fixed.

(b) Find the cost of 12 boards. [1 mark]

$C=17(12)+36=240$.

Answer: AUD 240.

2. Adult and concession tickets total 35. Adult tickets cost AUD 14 and concessions AUD 9; total revenue is AUD 390. [5 marks]

(a) Form and solve simultaneous equations. [4 marks]

$a+c=35$, $14a+9c=390$.

Substitution gives $5a+315=390$, so a=15 and c=20.

Answer: 15 adults and 20 concessions.

(b) Check the answer in context. [1 mark]

$14(15)+9(20)=210+180=390$.

Answer: Counts sum to 35 and revenue is AUD 390.

3. A subscription costs AUD 120 and AUD 2 per visit. Casual visits cost AUD 8 each. [4 marks]

(a) Find the equal-cost number of visits. [2 marks]

$120 + 2n = 8n$ gives n=20.

Answer: 20 visits.

(b) Find the first whole-number visit count at which the subscription is cheaper. [2 marks]

Strictly cheaper requires $120 + 2n < 8n$, hence n>20.

Answer: 21 visits.

4. Solve $4(3 - x) \geq 2x - 6$. [4 marks]

(a) Find the solution set. [3 marks]

$12 - 4x \geq 2x - 6$ gives $18 \geq 6x$.

Answer: $x \leq 3$.

(b) Explain whether the boundary is included. [1 mark]

The non-strict inequality allows equality at x=3.

Answer: Yes.

5. A workshop makes x stands and y shelves. Each stand needs 3 planks and each shelf 2. At most 31 planks are available, and no more than 8 shelves can be sold. [4 marks]

(a) Write all restrictions, including the nature of the variables. [3 marks]

Resources and demand are distinct restrictions.

Answer: $3x + 2y \leq 31$, $y \leq 8$, x,y non-negative integers.

(b) Test 5 stands and 8 shelves. [1 mark]

$3(5) + 2(8) = 31$.

Answer: Feasible; all 31 planks are used.

6. A feasible polygon has vertices $(0, 0), (10, 0), (6, 6), (0, 9)$. The objective is $P = 6x + 4y$. [5 marks]

(a) Evaluate P and identify every optimal vertex. [3 marks]

Substitute the four coordinates.

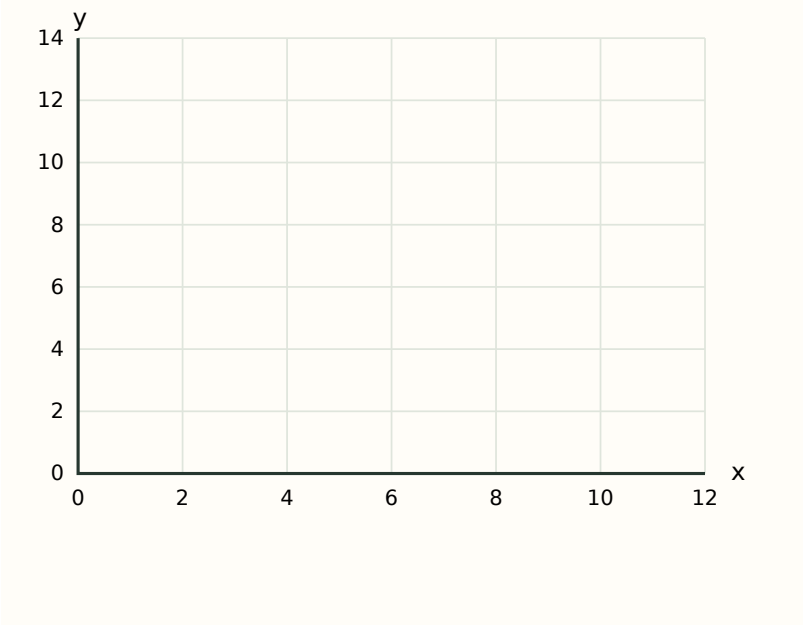
Answer: Values 0,60,60,36; (10,0) and (6,6) tie.

(b) Describe all continuous optima, not just vertices. [2 marks]

A linear objective constant at both ends of an edge is constant throughout that edge.

Answer: Every point on the edge between (10,0) and (6,6).

7. A printer produces x posters and y banners. There are 20 sheet-units and 24 machine-units. A poster uses 2 sheet-units and 1 machine-unit; a banner uses 1 and 2. Profits are AUD 5 and AUD 6. Counts must be whole numbers. [8 marks]



(a) State the model, including variable restrictions. [3 marks]

Define x and y as described.

Translate each separate resource or minimum requirement.

Answer: $2x + y \leq 20$, $x + 2y \leq 24$, non-negative integer x,y; Maximise $P = 5x + 6y$.

(b) Find and justify the optimal decision. Give the objective value. [5 marks]

Feasible vertices: (0, 0), (0, 12), (16/3, 28/3), (10, 0).

Check permitted whole-number decisions in the bounded region; for each x, compare the feasible endpoint y-values.

For x=0,1,2,3,4,5,6,7,8,9,10, the largest allowed y-values are 12,11,11,10,10,9,8,6,4,2,0. These give objectives 72,71,76,75,80,79,78,71,64,57,50 respectively. No integer x above 10 is feasible. The maximum is 80 at (4,10).

Answer: (4, 10); objective 80.

8. A food stall needs at least 70 servings. A small tray holds 12 servings and costs AUD 18; a large tray holds 20 and costs AUD 27. Only 5 trays fit in the warmer. [7 marks]

(a) Form a minimum-cost integer model. [3 marks]

Define x as small trays and y as large trays.

Answer: Minimise $C = 18x + 27y$ with $12x + 20y \geq 70$, $x + y \leq 5$, non-negative integers.

(b) Find the least cost and check all possible large-tray counts. [4 marks]

For y=0 no 5-tray plan suffices. For y=1 the required 5 small trays exceed the maximum of five trays.

For y=2 need x=3, cost 108; for y=3 need x=1, cost 99; y=4 costs 108 and y=5 costs 135.

One small plus three large trays provides 72 servings in 4 trays; no candidate costs less.

Answer: 1 small and 3 large trays; AUD 99.

9. A continuous solver recommends $(4.5, 2.5)$ for indivisible products. A student proposes $(5, 3)$ without checking. [4 marks]

(a) Explain why the rounded recommendation is not justified. [2 marks]

The continuous optimum is not an integer feasibility proof.

Answer: It may violate resource limits and may not be the best integer point.

(b) Describe a systematic method for a small bounded model. [2 marks]

Bounds make the search finite.

Answer: Enumerate allowed integer x-values and feasible endpoint y-values, evaluate the objective and verify the best pair.

10. A model maximises profit using only material constraints. One product also requires specialist staff available for 6 hours. [4 marks]

(a) Explain the missing information needed to add a staffing constraint. [2 marks]

A resource bound needs both per-unit coefficients and total availability.

Answer: The specialist hours required per unit of each product.

(b) Would adding a staffing restriction ever increase the maximum profit, with all other data fixed? [2 marks]

It removes feasible choices; the new maximum cannot exceed the old maximum.

Answer: No.