Linear models and optimisation - Practice test 4

10 questions | 44 marks | 70 minutes

Interpretation and error analysis, including empty, unbounded and non-unique solutions.

Equivalent correct methods earn the corresponding marks.

1. A transport model is $C = 75 + 1.8d$, with C in dollars and d in kilometres. [4 marks]

(a) Interpret the gradient and intercept. [2 marks]

The gradient is the rate per distance; the intercept is the zero-distance charge.

Answer: AUD 1.80 per kilometre and AUD 75 fixed.

(b) Find d when the quoted charge is AUD 156. [2 marks]

$1.8d = 156 - 75 = 81$, so d=45.

Answer: 45 km.

2. Consider $2x + 3y = 12$ and $4x + 6y = k$. [4 marks]

(a) For what k are there infinitely many solutions? [2 marks]

The second equation must be twice the first.

Answer: k=24.

(b) Explain the number of solutions if k=25. [2 marks]

The parallel boundary lines have inconsistent constants.

Answer: No solution.

3. A learner claims that solving $9 - 3x \leq 0$ gives $x \leq 3$. [3 marks]

(a) Find and explain the error. [2 marks]

$-3x \leq -9$ implies $x \geq 3$.

Answer: Division by a negative should reverse the symbol.

(b) Use a numerical counterexample to disprove the learner's result. [1 mark]

$9 - 3(0) = 9$, which is not at most 0.

Answer: For example x=0 satisfies their answer but not the original inequality.

4. A proposed feasible region is formed by shading every point that satisfies at least one of $x + y \leq 6$ and $2x + y \leq 8$. [4 marks]

(a) Explain the modelling error. [2 marks]

Meeting one resource limit does not excuse violating another.

Answer: All constraints must hold together; use intersection, not union.

(b) Test $(5, 0)$ against both inequalities. [2 marks]

Its sums are 5 and 10;10 exceeds 8.

Answer: It satisfies the first but fails the second.

5. The constraints are $x, y \geq 0$, $x + y \leq 5$ and $x + y \geq 8$. [3 marks]

(a) Explain whether a feasible solution exists. [2 marks]

No real sum can be at most 5 and at least 8.

Answer: No; the sum bounds contradict one another.

(b) Can a different objective repair the model? [1 mark]

The objective cannot create feasible points.

Answer: No.

6. The feasible set is $x, y \geq 0$ and $x + y \geq 6$. [5 marks]

(a) Minimise $C = 2x + 5y$. [3 marks]

Rewrite $C = 2(x + y) + 3y$.

The constraints give $C \geq 12$, with equality at (6,0).

Answer: Minimum 12 at(6,0).

(b) Maximise the same C. [2 marks]

At(t,0), C=2 t grows without bound for t at least 6.

Answer: No finite maximum.

7. For $x, y \geq 0$ and $x + y \leq 8$, consider $P = 3x + 3y$. [5 marks]

(a) State the maximum and all continuous optimal points. [3 marks]

$P = 3(x + y) \leq 24$, with equality exactly on that edge.

Answer: Maximum 24 at every point on x+y=8 in the first quadrant.

(b) How many whole-number optimal pairs exist? [2 marks]

x can be 0 through 8, with y=8-x.

Answer: 9.

8. A region has vertices $(0, 0), (6, 0), (4, 4), (0, 6)$. A student evaluates $P = 7x + 5y$ only at the largest x-value. [4 marks]

(a) Find the correct optimum. [3 marks]

Vertex values are 0,42,48,30.

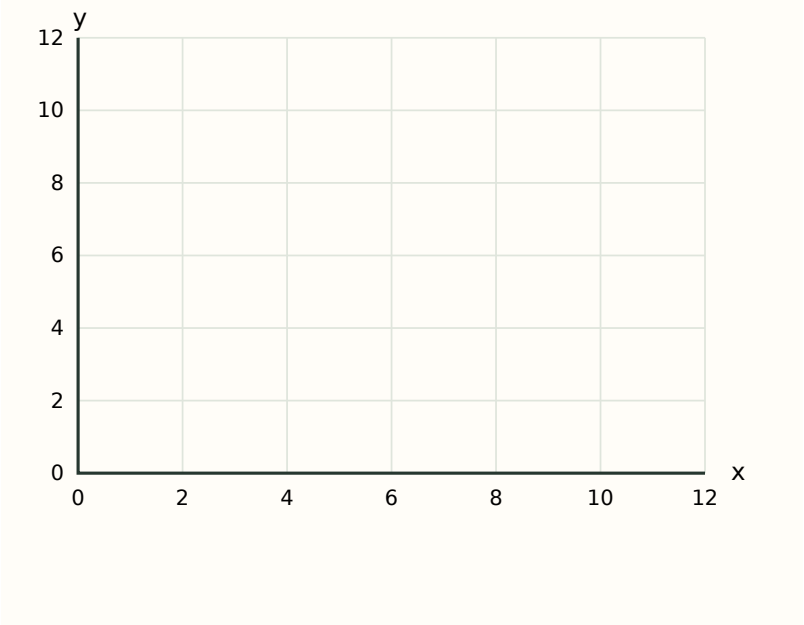
Answer: 48 at(4,4).

(b) Explain why the student's strategy fails. [1 mark]

The greatest x-coordinate need not give the greatest weighted total.

Answer: The objective rewards both variables.

9. A café prepares x cold packs and y warm packs. Space allows x+y at most 9; preparation allows x+2 y at most 14. Profits are AUD 3 and AUD 5. [8 marks]



(a) State the model, including variable restrictions. [3 marks]

Define x and y as described.

Translate each separate resource or minimum requirement.

Answer: $x + y \leq 9$, $x + 2y \leq 14$, $x, y \geq 0$; Maximise $P = 3x + 5y$. Counts x and y must be integers.

(b) Find and justify the optimal decision. Give the objective value. [5 marks]

Feasible vertices: (0, 0), (0, 7), (4, 5), (9, 0).

(0, 0) gives 0; (0, 7) gives 35; (4, 5) gives 37; (9, 0) gives 27

The largest vertex value is 37.

The continuous optimum has whole-number coordinates, so it also gives the optimum for the item counts.

Answer: (4, 5); objective 37.

10. A report says: “Our model proves that the business will earn exactly AUD 2,000 next week.” Its model assumes constant sales prices and that every unit is sold. [4 marks]

(a) Rewrite the conclusion so it is justified. [2 marks]

An optimised model is conditional on the inputs and assumptions.

Answer: The model predicts AUD 2,000 if its stated assumptions and data hold.

(b) Name two sources of uncertainty relevant to that prediction. [2 marks]

Actual sales or discounts can change realised profit.

Answer: Demand and selling price, for example.