Linear models and optimisation - Practice test 5

10 questions | 49 marks | 70 minutes

Applied decisions with profit, minimum requirements and sensitivity.

Equivalent correct methods earn the corresponding marks.

1. A community event pays AUD 160 for a printer setup and AUD 1.20 for each programme. [4 marks]

(a) Write total cost for n programmes and find the cost of 300. [2 marks]

Substitute n=300 to obtain 160+360.

Answer: $C = 160 + 1.20n$; AUD 520.

(b) Explain how a fixed AUD 40 delivery fee changes the rule. [2 marks]

Delivery adds the same amount at every quantity.

Answer: $C = 200 + 1.20n$; the gradient is unchanged.

2. An event sells 70 tickets: adult tickets are AUD 18 and student tickets AUD 12. Revenue is AUD 1,020. [5 marks]

(a) Find the two ticket counts. [4 marks]

$a + s = 70$, $18a + 12s = 1020$.

Substitute s=70-a to get 6 a+840=1020.

Answer: 30 adult and 40 student.

(b) Explain why ticket counts need integer and non-negative conditions. [1 mark]

A fractional or negative ticket count is not a practical solution.

Answer: They count indivisible people.

3. A normal pack sells for AUD 24 and costs AUD 15. A premium pack sells for AUD 34 and costs AUD 20. There is also an AUD 80 fixed operating cost. [4 marks]

(a) Write total profit for x normal and y premium packs. [3 marks]

Unit contributions are 9 and 14; subtract the fixed cost once.

Answer: $P = 9x + 14y - 80$.

(b) Find profit for 10 normal and 8 premium packs. [1 mark]

$90 + 112 - 80 = 122$.

Answer: AUD 122.

4. Each normal pack uses 3 printing units and 2 packing units. Each premium pack uses 2 printing units and 4 packing units. There are 96 printing units and 112 packing units. [5 marks]

(a) Form the resource inequalities. [3 marks]

Use a separate inequality for each resource.

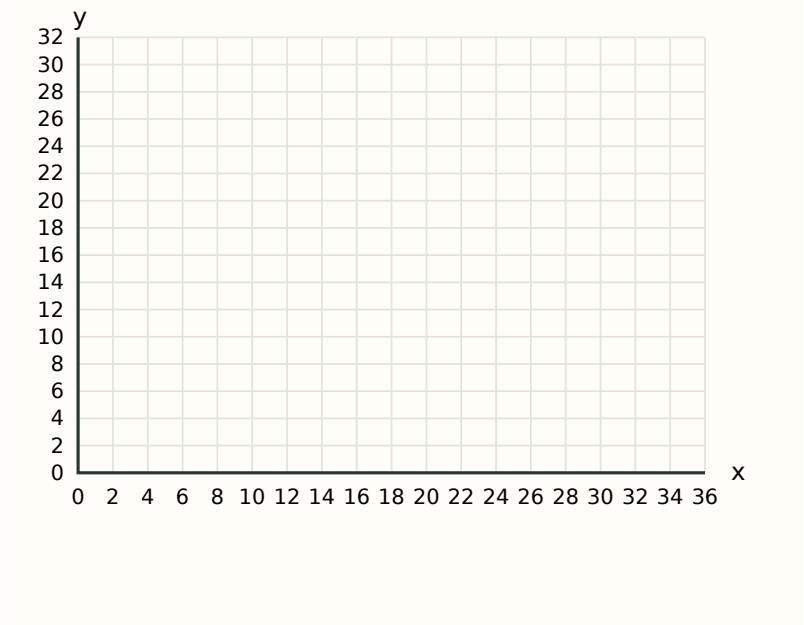
Answer: $3x + 2y \le 96$, $2x + 4y \le 112$, $x, y \ge 0$.

(b) Is the plan $(20, 16)$ feasible? [2 marks]

Both uses are within the limits.

Answer: Yes; it uses 92 printing and 104 packing units.

5. For the restrictions in Question 4, find the continuous plan maximising $9x + 14y$. [6 marks]



(a) Find the feasible vertices. [3 marks]

Solve the two active resource equations: doubling the first and subtracting the second gives 4 x=80.

Answer: $(0, 0), (32, 0), (20, 18), (0, 28)$.

(b) Find the maximum contribution and the profit after the fixed AUD 80 cost. [3 marks]

Vertex values are 0,288,432,392.

Subtract 80 from 432.

Answer: Maximum contribution 432 at(20,18); profit AUD 352.

6. A caterer needs at least 18 kilograms of a blend and at least 30 nutrient units. A has 1 unit/kg and B has 3 units/kg. Costs are AUD 4/kg for A and AUD 7/kg for B. [7 marks]

(a) Form a continuous minimum-cost model. [3 marks]

Define x and y as kilograms of A and B.

Answer: Minimise $C = 4x + 7y$ with $x + y \geq 18$, $x + 3y \geq 30$, $x, y \geq 0$.

(b) Find and justify the optimum. [4 marks]

Lower corners are(30,0),(12,6),(0,18).

Costs are 120,90,126; the positive cost increases away from the lower boundary.

Answer: 12 kg of A and 6 kg of B; AUD 90.

7. A club books minivans with 8 seats costing AUD 75 and coaches with 20 seats costing AUD 160. At least 44 seats are required and at most 5 vehicles can be booked. [7 marks]

(a) Form an integer cost model. [3 marks]

Keep seats and vehicle-count bounds separate.

Answer: Minimise $C = 75x + 160y$, subject to $8x + 20y \geq 44$, $x + y \leq 5$, non-negative integers.

(b) Find the cheapest booking. [4 marks]

With no coach, five minivans give only 40 seats.

For y=1 require x=3, cost 385; y=2 require x=1, cost 395; y=3 costs 480 and larger y cost more.

The chosen plan gives 44 seats in 4 vehicles.

Answer: 3 minivans and 1 coach, cost AUD 385.

8. An optimisation model already has $x, y \geq 0$ and $x + y \leq 10$. Someone adds $x \leq 12$. [3 marks]

(a) Explain whether the added constraint changes the feasible region. [2 marks]

Non-negativity and x+y at most 10 already imply x at most 10.

Answer: No; it is redundant.

(b) Can a redundant constraint change the optimum for an unchanged objective? [1 mark]

The feasible set remains exactly the same.

Answer: No.

9. A feasible triangle has vertices $(0, 0), (10, 0), (0, 8)$. Product profits are p for x and 6 for y, where p is positive. [4 marks]

(a) Find when the two non-zero vertices have equal profit. [2 marks]

$10p = 6(8) = 48$, so p=4.8.

Answer: $p = 4.8$.

(b) Describe the optimum if p=5 and if p=4. [2 marks]

Compare 10 p with 48.

Answer: At p=5 choose(10,0), profit 50; at p=4 choose(0,8), profit 48.

10. The optimal plan uses 18 premium packs, but the supplier sells packing materials only in batches sufficient for 5 premium packs. [4 marks]

(a) Explain why a new practical restriction may be needed. [2 marks]

A continuous or arbitrary integer quantity may not represent how materials are purchased.

Answer: The model may need batch-size conditions or a revised cost structure.

(b) What information is needed before changing the recommendation? [2 marks]

Do not assume all unused material is wasted or free without information.

Answer: Batch prices, whether leftover material can be stored, and any minimum-order conditions.