Transition matrices and state vectors

6 questions | 20 marks

Equivalent correct methods earn the corresponding marks.

1. Use rows (0.7,0.2),(0.3,0.8) to update (80,20). [3 marks]

A: $0.7(80) + 0.2(20) = 60$.

B: $0.3(80) + 0.8(20) = 40$.

Answer: (60,40).

2. Rows of a proposed closed transition matrix are (0.4,0.3),(0.5,0.7). Is it valid? [3 marks]

It does not account for 10% of those initially in A.

Answer: No; the first column totals 0.9.

3. Find a, b so rows (0.75,a),(b,0.6) form a column-stochastic matrix. [3 marks]

$0.75 + b = 1$ and $a + 0.6 = 1$.

Answer: $a = 0.4, b = 0.25$.

4. A state vector is (120,80). What total must a closed transition preserve? [3 marks]

The starting entries sum to 200.

Column sums of 1 preserve this total.

Answer: 200.

5. Each day 90% of North bikes stay North; 40% of South bikes move North. [5 marks]

(a) Write the North,South column-stochastic matrix. [2 marks]

Each column describes destinations from one depot.

Answer: Rows (0.9,0.4),(0.1,0.6).

(b) Predict tomorrow from (20,30). [3 marks]

North: $0.9(20) + 0.4(30) = 30$.

South: $0.1(20) + 0.6(30) = 20$.

Answer: (30,20).

6. A forecast is (34.6,15.4) for 50 bikes. Explain its meaning. [3 marks]

An operational integer plan should still total 50.

Do not interpret fractional counts literally.

Answer: It is an expected distribution; actual whole-bike counts vary.