Long-term behaviour and steady states

6 questions | 19 marks

Equivalent correct methods earn the corresponding marks.

1. For rows (0.8,0.3),(0.2,0.7), find steady proportions. [3 marks]

$0.2a = 0.3b$ and $a + b = 1$.

Answer: (0.6,0.4).

2. Use the same model for 350 members. Find steady counts. [3 marks]

Multiply (0.6,0.4) by 350.

Answer: (210,140).

3. For rows (0.9,0.4),(0.1,0.6), find steady proportions. [3 marks]

$0.1a = 0.4b$ and $a + b = 1$.

Answer: (0.8,0.2).

4. For the swap matrix, find steady proportions and explain the behaviour from (1,0). [3 marks]

Balance gives $a = b$.

Applying the swap repeatedly never approaches that balance from (1,0).

Answer: Steady (0.5,0.5); the specified start alternates (1,0),(0,1).

5. Find steady proportions for rows (0.5,0.25,0),(0.5,0.5,0.5),(0,0.25,0.5). [3 marks]

The first and third equations give $a = c = 0.5b$.

Normalise using $2b = 1$.

Answer: (0.25,0.5,0.25).

6. A closed model has steady counts (80,120) out of 200. [4 marks]

(a) State the steady proportions. [1 mark]

Divide by 200.

Answer: (0.4,0.6).

(b) Predict steady counts if the total doubles and the transition rule is unchanged. [2 marks]

Multiply the proportions by 400.

Answer: (160,240).

(c) Give one reason observed long-run counts might differ. [1 mark]

The equilibrium prediction is conditional on its assumptions.

Answer: The system may open or transition behaviour may change.