Matrix models - Practice test 1

10 questions | 45 marks | 70 minutes

Core techniques, network interpretation and two-state forecasts. Network matrices use rows=start and columns=destination; transition matrices use columns=current and rows=next unless stated otherwise.

Equivalent correct methods earn the corresponding marks.

1. Matrices have rows A: $(2,-1),(3,4)$, and B: $(1,5),(-2,0)$. [5 marks]

(a) Find $A+B$. [2 marks]

Add corresponding entries.

Answer: Rows $(3,4),(1,4)$.

(b) Find $2A-B$. [3 marks]

Double A, then subtract corresponding entries of B.

Answer: Rows $(3,-7),(8,8)$.

2. A shop sells two products at AUD 4 and AUD 7. Quantities sold on three days are $(5,2),(3,4),(6,1)$. [5 marks]

(a) State the orders of the sales matrix and price vector. [2 marks]

Rows represent days; columns represent products.

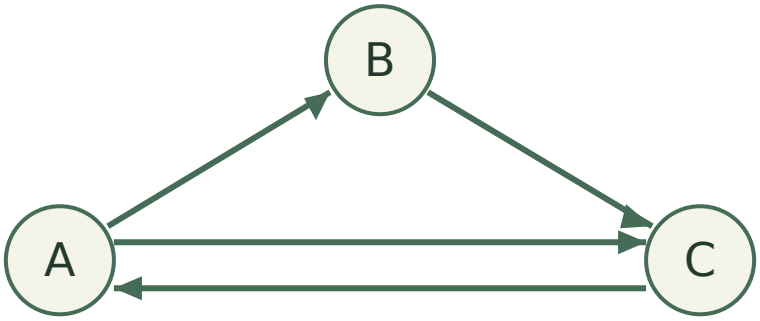
Answer: 3×2 and 2×1.

(b) Use a matrix product to calculate the three daily incomes. [3 marks]

Compute $5(4)+2(7)=34$, $3(4)+4(7)=40$, $6(4)+1(7)=31$.

Answer: Column $(34,40,31)$ dollars.

3. The diagram shows direct links, ordered A,B,C. [4 marks]



Each arrow is one permitted move.

(a) Write the adjacency matrix. [3 marks]

Record every arrow in its starting row and destination column.

Answer: [(0, 1, 1); (0, 0, 1); (1, 0, 0)]

(b) State the number of links entering C. [1 mark]

Sum column C.

Answer: 2.

4. Use adjacency rows (0,1,1),(0,0,1),(1,0,0), ordered A,B,C. [5 marks]

(a) Calculate A^2. [3 marks]

Row 1 entries: $0(0)+1(0)+1(1)=1$; $0(1)+1(0)+1(0)=0$; $0(1)+1(1)+1(0)=1$.

Row 2 entries: $0(0)+0(0)+1(1)=1$; $0(1)+0(0)+1(0)=0$; $0(1)+0(1)+1(0)=0$.

Row 3 entries: $1(0)+0(0)+0(1)=0$; $1(1)+0(0)+0(0)=1$; $1(1)+0(1)+0(0)=1$.

So A^2 has rows (1,0,1),(1,0,0),(0,1,1).

Answer: [(1, 0, 1); (1, 0, 0); (0, 1, 1)]

(b) Interpret entry row A, column C of A^2 and identify the walk. [2 marks]

The entry equals 1 and counts exactly two steps.

Answer: One two-step walk: A→B→C.

5. An unweighted directed network is the chain P→Q→R→S. [4 marks]

(a) Is S reachable from P? Which matrix power first records it? [2 marks]

The shortest route uses three edges.

Answer: Yes; A^3.

(b) Can S reach P? Explain. [2 marks]

No arrow points back along the chain.

Answer: No.

6. A beats B and C; B beats C; C has no wins. Score by row sums of $D + 0.5D^2$. [5 marks]

(a) Calculate the direct-win scores. [2 marks]

Count the outgoing dominance arrows.

Answer: (2,1,0).

(b) Calculate the weighted scores and rank the teams. [3 marks]

Only A→B→C adds a two-step score of 0.5.

Answer: (2.5,1,0); A, B, C.

7. A two-state transition matrix has rows (0.8,0.3),(0.2,0.7), in A,B order. [4 marks]

(a) Interpret the entry 0.3. [1 mark]

It is destination row A, starting column B.

Answer: 30% of those currently in B move to A next period.

(b) Predict the next state from (120,80). [3 marks]

A: $0.8(120) + 0.3(80) = 120$.

B: $0.2(120) + 0.7(80) = 80$.

Answer: (120,80).

8. A proposed closed transition matrix has rows $(0.6,p),(q,0.75)$. [4 marks]

(a) Find p, q. [2 marks]

Set each column sum equal to 1.

Answer: $p = 0.25, q = 0.4$.

(b) Is column summation alone sufficient to check a transition matrix? Explain. [2 marks]

Negative entries can still occur in columns totalling 1.

Answer: No; all entries must also be probabilities between 0 and 1.

9. For T with rows $(0.8,0.3),(0.2,0.7)$, let $x_0 = (100, 0)$. [4 marks]

(a) Find x_3. [3 marks]

The successive states are $(80,20),(70,30),(65,35)$.

Answer: $(65,35)$.

(b) Explain why T^3 alone is not the population forecast. [1 mark]

The population depends on the starting distribution.

Answer: It is a transition matrix; multiply by the initial vector.

10. A bike model uses rows $(0.7,0.2),(0.3,0.8)$. There are 250 bikes. [5 marks]

(a) Find steady counts. [3 marks]

Balance gives $0.3a = 0.2b$, hence ratio 2:3.

Use total 250.

Answer: $(100,150)$.

(b) Give two assumptions needed for using this as a long-run forecast. [2 marks]

Also assess whether the model approaches this equilibrium.

Answer: The bike population remains closed and the transition rule stays appropriate each day.