Matrix models - Practice test 3

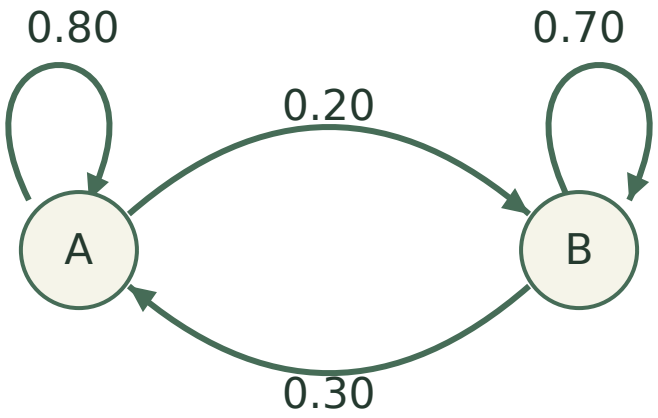
Name: _____ Date: _____

10 questions | 56 marks | 70 minutes

Transition estimation, three-state balance and forecast comparison. Network matrices use rows=start and columns=destination; transition matrices use columns=current and rows=next unless stated otherwise.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. Use the two-state transition diagram. [5 marks]



(a) Write its transition matrix in A,B order with column states. [3 marks]

(b) Explain why adjacency and transition conventions must not be silently interchanged. [2 marks]

2. A survey reports proportions (0.25,0.75) for a group of 800. Rows of T are (0.8,0.3),(0.2,0.7). [4 marks]

(a) Write the current count vector. [1 mark]

(b) Predict the next proportions and counts. [3 marks]

3. A month's movements are supplied below. [6 marks]

Observed transitions: rows=destination, columns=start.

Destination / start	A	B
A	80	15
B	20	35

(a) Estimate a transition matrix using starting-state totals. [3 marks]

(b) Predict next month from the final state of this month. [3 marks]

4. A three-state model has transition rows (0.5,0.25,0),(0.5,0.5,0.5),(0,0.25,0.5). [6 marks]

(a) Verify it is column-stochastic. [2 marks]

(b) Predict two periods from (200,0,0). [4 marks]

5. For rows T: (0.7,0.2),(0.3,0.8), [6 marks]

(a) Calculate T^2. [3 marks]

(b) Interpret its entry in row A,column B, then predict two periods from (0,200). [3 marks]

6. Use the three-state matrix in Question 4, with a total of 320. [6 marks]

(a) Find steady counts. [4 marks]

(b) Verify your state by multiplication. [2 marks]

7. A maintenance firm has current state (160,40). Rows of T are (0.8,0.3),(0.2,0.7). Next-period cost per item is AUD 4 in A and AUD 7 in B. [6 marks]

(a) Predict the next state. [3 marks]

(b) Find expected next-period cost and compare with applying the same costs to today's state. [3 marks]

8. A 500-member model uses rows (0.8,0.3),(0.2,0.7), starting at (500,0). [6 marks]

(a) Calculate the state after three periods. [3 marks]

(b) Find steady counts and explain why they differ from the third-period forecast. [3 marks]

9. Compare fast matrix F: (0.8,0.3),(0.2,0.7), and slow matrix S: (0.98,0.03),(0.02,0.97), starting at (100,0). [7 marks]

(a) Find the steady proportions of both. [3 marks]

(b) Use technology to find the A amount after five periods and explain the difference. [4 marks]

10. Two held-out observations are supplied below. [4 marks]

Observed and predicted counts.

Month	Prediction A	Prediction B	Observed A	Observed B
1	70	30	74	26
2	65	35	66	34

(a) Find the maximum absolute state error in each month. [2 marks]

(b) Explain why fitting exactly to month 1 might worsen model evaluation. [2 marks]