# Matrix models - Practice test 4

Name: _____ Date: _____

10 questions | 51 marks | 70 minutes

Structure, exceptional behaviour and careful interpretation. Network matrices use rows=start and columns=destination; transition matrices use columns=current and rows=next unless stated otherwise.

Use a graphics calculator or appropriate digital tool for labelled tasks.

**1.** Let $A$ have rows (1,2),(0,1), and let $I$ be the identity matrix. [5 marks]

**(a)** Find $AI$ and $IA$. [2 marks]

**(b)** Calculate $A^2$ and explain why it is not $2A$. [3 marks]

**2.** A row-start adjacency matrix $C$ has rows (0,1,0),(0,0,1),(1,0,0). [5 marks]

**(a)** Calculate $C^3$. [3 marks]

**(b)** Explain why all different vertices are reachable although $(C^3)_{AB} = 0$. [2 marks]

**3.** A two-vertex network has a loop at each vertex and an arrow A→B. Its adjacency rows are (1,1),(0,1). [5 marks]

**(a)** Find the number of two-step A-to-B walks and list them. [3 marks]

**(b)** How many three-step A-to-B walks are there? [2 marks]

**4.** A dominance matrix has rows (0,1,1),(0,0,1),(0,0,0). A score is the row sum of $D + wD^2$. [5 marks]

**(a)** Express the three scores in terms of $w$. [3 marks]

**(b)** For nonnegative $w$, can B overtake A under this rule? Explain. [2 marks]

**5.** A transition matrix is the $2 \times 2$ identity. [4 marks]

**(a)** Find all steady states totalling 100. [2 marks]

**(b)** Does the model predict a unique long-run split independent of the start? [2 marks]

**6.** A transition matrix has rows (0,1),(1,0), and initial state (80,20). [5 marks]

**(a)** Find $x_{10}$ and $x_{11}$. [2 marks]

**(b)** Find a steady state with the same total. Does this initial state approach it? [3 marks]

**7.** Let $T$ have rows (0.8,0.3),(0.2,0.7), let $S$ swap A and B, and start at (80,20). [6 marks]

**(a)** Apply $T$ first, then $S$. [3 marks]

**(b)** Apply $S$ first, then $T$, and compare. [3 marks]

**8.** A closed 100-person group uses rows (0.8,0.3),(0.2,0.7). The next state is (68,32). [6 marks]

**(a)** Recover the initial state. [4 marks]

**(b)** Verify it by multiplication. [2 marks]

**9.** An open model first applies rows (0.8,0.3),(0.2,0.7), then adds 5 new members to A. Start at (60,40). [5 marks]

**(a)** Find the next state and total. [3 marks]

**(b)** Explain why writing only $x_1 = Tx_0$ would be inadequate. [2 marks]

**10.** A report claims: “If every column sums to 1, the model is valid and every starting state approaches one accurate prediction.” [5 marks]

**(a)** Give two different mathematical objections. [4 marks]

**(b)** Give one empirical objection. [1 mark]