Matrix models - Applications test 6

Name: _____ Date: _____

10 questions | 59 marks | 70 minutes

Linked models, inference and justified recommendations. Network matrices use rows=start and columns=destination; transition matrices use columns=current and rows=next unless stated otherwise.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. A closed two-state matrix has rows $(0.9, b), (0.1, 1-b)$. A current state (60,40) is predicted to become (62,38). [6 marks]

(a) Find b and check the resulting matrix. [3 marks]

(b) Find steady counts for a total of 300. [3 marks]

2. A service-count adjacency matrix has rows $(0,k,1),(0,0,2),(1,0,0)$, ordered A,B,C, with k a nonnegative integer. [6 marks]

(a) If there are six two-step walks A to C, find k. [3 marks]

(b) Find the number of three-step A-to-C walks and explain the route. [3 marks]

3. Three towns form the directed cycle A→B→C→A. [5 marks]

(a) Find which of $A_{AB}, (A^2)_{AB}, (A^3)_{AB}, (A^4)_{AB}$ are nonzero. [3 marks]

(b) Explain why a route-planning app must distinguish reachability from a required exact number of transfers. [2 marks]

4. A ranking rule counts direct wins and two-step wins. Teams A,B,C form the cycle A beats B, B beats C, C beats A. [5 marks]

(a) Show that the rule cannot select a unique winner if all direct wins get weight 1 and all two-step wins the same weight w. [3 marks]

(b) Give two pieces of additional evidence that could improve a ranking model without merely inventing a tie-break. [2 marks]

5. A 100-member population uses transition rows (0.9,0.4),(0.1,0.6). The next state is (60,40). [6 marks]

(a) Recover the current state. [4 marks]

(b) Verify both destination counts. [2 marks]

6. Start at (100,0). Let T have rows (0.8,0.3),(0.2,0.7), and B rows (0.7,0.2),(0.3,0.8). [7 marks]

(a) Apply policies T, then B, then T. Write the product and final state. [4 marks]

(b) Compare the final state for policies B, then T, then B. [3 marks]

7. A three-state matrix has rows $(0.5,a,0),(0.5,1-2a,0.5),(0,a,0.5)$. [5 marks]

(a) Find the allowable range of a for a transition matrix. [2 marks]

(b) Find a if (25,50,25) is steady. [3 marks]

8. A two-state policy keeps 85% of current A in A. The proportion of B moving to A is b. [7 marks]

(a) Choose b so the steady A proportion is 0.7. [4 marks]

(b) If b rises to 0.4, find the new steady A proportion and interpret. [3 marks]

9. A model uses rows (0.8,0.3),(0.2,0.7), initially (200,80). The observed next state is (180,100). [5 marks]

(a) Find the one-period prediction and maximum absolute error. [3 marks]

(b) The model’s two-period prediction is exactly the later observation (176,104). Does this validate the model permanently? [2 marks]

10. Options F and S use transition rows (0.8,0.3),(0.2,0.7) and (0.98,0.03),(0.02,0.97), respectively. Start at (100,0). Per-period service cost is AUD 5 for A and AUD 1 for B. [7 marks]

(a) Compare expected costs using the distribution after three periods. [4 marks]

(b) Compare steady-state costs and explain why the preferred short-term option need not follow from equilibrium alone. [3 marks]