Least-squares regression

6 questions | 18 marks

Equivalent correct methods earn the corresponding marks.

1. Find the line from $\bar{x}=6$, $\bar{y}=20$, $S_{xx}=10$ and $S_{xy}=25$. [3 marks]

$b=25/10=2.5$.

$a=20-2.5\times6=5$.

Answer: $\hat{y}=5+2.5x$.

2. Given $r=-0.6$, $s_x=5$, $s_y=10$, $\bar{x}=8$ and $\bar{y}=30$, find the regression line. [3 marks]

$b=-0.6\times10/5=-1.2$.

$a=30-(-1.2\times8)=39.6$.

Answer: $\hat{y}=39.6-1.2x$.

3. A candidate line has residuals -2,1,1. Calculate its sum of squared residuals. [3 marks]

Square each residual: 4,1,1.

Add the squares, not their signed values.

Answer: 6.

4. Two models have residuals A=(-3,0,3) and B=(-2,1,1). Which has the smaller least-squares error? [3 marks]

A has squared sum 9+0+9=18.

B has 4+1+1=6, so B is better on this criterion.

Answer: B.

5. A regression line has a=7 and b=0.5. Predict y at x=12. [3 marks]

Substitute x=12 into $\hat{y}=7+0.5x$.

7+6=13.

Answer: 13.

6. A student rounds b=0.08473 to 0.1 before predicting at x=900. Explain the risk. [3 marks]

A small coefficient rounding can become large after multiplication by x.

Keep guard digits until the final prediction.

Answer: The slope contribution changes from 76.257 to 90, an error of 13.743.