Financial mathematics - practice test 2

10 questions | 60 marks | 75 minutes

Practice with reverse calculations, stepped contributions and a changing-rate loan.

Equivalent correct methods earn the corresponding marks.

1. A machine costs AUD 9,000 and loses 15% of its value annually. [6 marks]

(a) Find its modelled value after 4 years. [3 marks]

Use $9000(0.85)^4$.

Answer: AUD 4,698.06

(b) A seller claims its four-year value is $9000(1 - 4 \times 0.15)$. Explain the error. [3 marks]

Compound depreciation repeatedly multiplies by 0.85.

Answer: That is a simple linear reduction, not 15% of each year’s remaining value.

2. An effective annual rate is 5.5%. [6 marks]

(a) Find the equivalent monthly rate. [3 marks]

Solve $(1 + i)^{12} = 1.055$.

Answer: 0.4472%

(b) Find its equivalent nominal annual quotation with monthly compounding. [3 marks]

Multiply the exact monthly rate by 12.

Answer: 5.3660%

3. AUD 2,500 is saved initially, followed by AUD 600 at each year end for 8 years at 4%. [6 marks]

(a) Find the final balance. [3 marks]

Combine $2500(1.04)^8$ and $600s_{\overline{8}|}(0.04)$.

Answer: AUD 8,949.96

(b) Separate total money deposited from interest. [3 marks]

Deposits is $2500 + 8 \times 600$.

Subtract deposits from the final balance.

Answer: Deposited:AUD 7,300.00; interest:AUD 1,649.96.

4. Starting from zero, AUD 900 is deposited at each year end at 3%. The goal is AUD 7,000. [6 marks]

(a) Find the first whole number of deposits that reaches the goal. [3 marks]

Solven is $\ln(1 + 7000 \times 0.03/900)/\ln(1.03)$, then round up.

Answer: 8

(b) Verify the crossing using balances before and at that deposit. [3 marks]

The seventh balance is below 7000 and the eighth is above 7000.

Answer: After 7:AUD 6,896.22; after 8:AUD 8,003.10.

5. A fund starts at AUD 15,000. It earns 4% each year. Gross year-end contributions are AUD 4,000,4,200 and 4,400; a stated 10% deduction applies to each contribution. There are no fees. [6 marks]

Supplied contributions

Year	Gross contribution (AUD)
1	4000
2	4200
3	4400

(a) Find each net contribution. [3 marks]

Multiply each gross contribution by 0.9.

Answer: AUD 3,600; AUD 3,780; AUD 3,960.

(b) Find the balance after year 3. [3 marks]

Multiply the previous balance by 1.04, then add the current year’s net contribution, for each of the three years.

Answer: AUD 28,657.92

6. AUD 80,000 must provide 12 equal year-end withdrawals, earning 5% annually. [6 marks]

(a) Find the withdrawal amount. [3 marks]

Use $R = 80000/a_{12}(0.05)$.

Answer: AUD 9,026.03

(b) If the withdrawal rises by AUD 500, find the first closing balance. [3 marks]

Add one year’s interest, then subtract the increased withdrawal.

Answer: AUD 74,473.97

7. A family wants a purchase with today's price AUD 20,000 in 6 years. Prices are modelled to rise 2% annually. The savings account earns 4% annually. [6 marks]

(a) Find the future-dollar target. [3 marks]

Inflate today's price by 1.02^6.

Answer: AUD 22,523.25

(b) Find the equal year-end deposit needed, starting from zero. [3 marks]

Divide the future target *by* $s_6(0.04)$.

Answer: AUD 3,395.65

8. A fund needs AUD 12,000 at time 4 and earns 5% annually. Four equal year-end deposits were planned, but the time 2 deposit is missed. [6 marks]

(a) Calculate the originally planned deposit R. [3 marks]

Use $R = 12000/s_4(0.05)$.

Answer: AUD 2,784.14

(b) The original deposits at times 1,3 and 4 are paid. How much extra at time 4 replaces the missed time 2 deposit? [3 marks]

The omitted deposit would have earned for two years; extra is $R(1.05)^2$.

Answer: AUD 3,069.52

9. A loan of AUD 18,000 is to be cleared in 48 monthly payments. Interest is 0.5% per month. Use the unrounded exact repayment throughout. [6 marks]

(a) Find the monthly repayment. [3 marks]

Use $R = 18000/a_{48}(0.005)$.

Answer: AUD 422.73

(b) Find the outstanding balance after 18 payments. [3 marks]

Use B_{18} or discount the remaining 30 exact repayments.

Answer: AUD 11,749.39

10. A loan starts at AUD 20,000 with annual interest 5% and annual repayments AUD 5,000. After the second repayment the rate rises to 7%; it must then be cleared with three equal year-end repayments. [6 marks]

(a) Find the balance just after the second original repayment. [3 marks]

Apply the original recurrence for two years.

Answer: AUD 11,800.00

(b) Find the new annual repayment. [3 marks]

Treat that balance as a new three-year loan at 7%.

Use $R = B_2/a_3(0.07)$.

Answer: AUD 4,496.41