Financial mathematics - practice test 3

10 questions | 60 marks | 75 minutes

Payment timing, numerical rate solving and the effect of borrowing fees.

Equivalent correct methods earn the corresponding marks.

1. AUD 6,000 earns 0.4% per month for 18 months with no other cash flows. [6 marks]

(a) Find the final balance. [3 marks]

Use $6000(1.004)^{18}$.

Answer: AUD 6,447.01

(b) Find the nominal annual quotation and effective annual rate. [3 marks]

Nominal is $12 \times 0.4\%$.

Effective is $(1.004)^{12} - 1$.

Answer: Nominal 4.8%; effective 4.9070%.

2. Account A quotes 5.8% nominal compounded monthly. Account B quotes 5.9% nominal compounded quarterly. Both are fee-free. [6 marks]

(a) Find both effective annual rates. [3 marks]

Use $(1 + j/m)^m - 1$ for each account.

Answer: A:5.9567%; B:6.0318%.

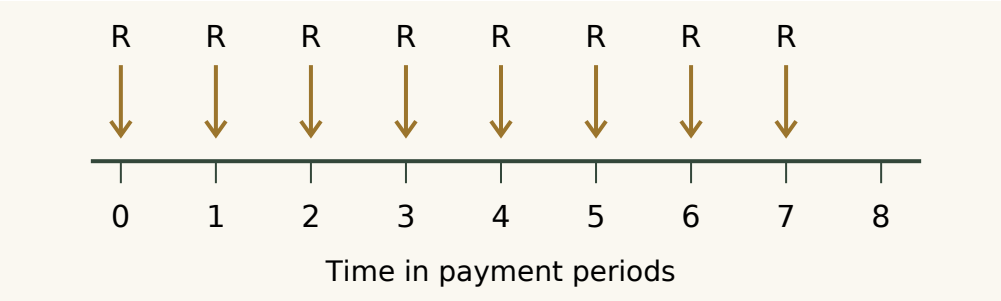
(b) Compare their one-year interest on AUD 30,000. [3 marks]

Multiply each exact effective rate by 30000.

B yields the higher one-year interest.

Answer: A:AUD 1,787.01; B:AUD 1,809.55.

3. Eight annual deposits of AUD 700 earn 4%. [6 marks]



(a) Find the balance at time 8 if deposits occur at times 0 through 7. [3 marks]

This is an annuity due; use $700s_{8}(0.04) \times 1.04$.

Answer: AUD 6,707.96

(b) Find how much more this is than deposits at times 1 through 8. [3 marks]

Subtract the ordinary annuity; the difference is 4% of its accumulated value.

Answer: AUD 258.00

4. Six end-of-year deposits of AUD 1,000 accumulate to AUD 6,800. Assume a constant rate between 0% and 10%. [6 marks]

(a) Find the annual rate numerically. [3 marks]

Solve $1000s_{6}(i) = 6800$ by numerical root search.

Answer: 4.9888%

(b) Check the result and calculate total interest. [3 marks]

Total deposits is 6000.

Interest is $6800 - 6000$.

Answer: The substitution returns AUD 6,800; interest is AUD 800.

5. Two funds start with AUD 40,000, earn 5% before fixed year-end fees, and receive net AUD 3,000 year-end contributions for 10 years. Their annual fees are AUD 100 and AUD 180. [6 marks]

(a) Find both final balances. [3 marks]

Use net year-end flows 2900 and 2820 respectively.

Answer: Fee 100:AUD 101,631.67; fee 180:AUD 100,625.44.

(b) Find the accumulated impact of the fee difference. [3 marks]

The difference is an annuity of AUD 80 per year accumulated at 5%.

Answer: AUD 1,006.23

6. An annuity must pay AUD 4,000 annually for 7 years at 6%. [6 marks]

(a) Find its present value if payments are at year ends. [3 marks]

Use $4000a_{7}(0.06)$.

Answer: AUD 22,329.53

(b) Find its present value if the first payment is immediate and six more follow annually. [3 marks]

The payments are all one period earlier; multiply by 1.06.

Answer: AUD 23,669.30

7. An investment earns 8% in a year. Tax takes 25% of the interest. A fee of 1% of the post-tax balance follows. Inflation is 3%. [6 marks]

(a) Find its net nominal growth factor. [3 marks]

After-tax interest rate is $0.08 \times 0.75 = 0.06$.

After-tax factor 1.06, then fee factor 0.99:$1.06 \times 0.99 = 1.0494$.

Answer: 1.0494

(b) Find the real annual return. [3 marks]

Divide 1.0494 by 1.03 and subtract 1.

Answer: 1.8835%

8. Compare repaying AUD 25,000 over 5 years. Model A:5% annual interest-only, plus a year-end sinking fund earning 3%. Model B:5% reducing-balance with five equal annual end repayments. Ignore fees. [6 marks]

(a) Find A's annual combined outflow. [3 marks]

Loan interest 1250 plus sinking deposit $25000/s_5(0.03)$.

Answer: AUD 5,958.86

(b) Find B's annual repayment and compare the total five-year cash outflows. [3 marks]

Calculate $B = 25000/a_5(0.05)$.

Multiply each regular outflow by 5; both models clear the debt at time 5.

Answer: B repayment:AUD 5,774.37; A total:AUD 29,794.32; B total:AUD 28,871.85.

9. AUD 8,000 is owed at 0.6% monthly. Compare month-end repayments AUD 400 and AUD 500; use exact balances and reduce the last payment. [6 marks]

(a) Find the payment count for each. [3 marks]

Iterate or solve the payoff equation and round the period count up.

Answer: AUD 400:22; AUD 500:17.

(b) Find the interest saved by choosing AUD 500. [3 marks]

Calculate actual final payments for both schedules.

Subtract their total interest; do not assume a full final repayment.

Answer: AUD 112.18

10. For an educational cost-rate model, a borrower receives AUD 4,900 after a fee and pays AUD 450 at each month end for 12 months. [6 marks]

(a) Write the equation for the monthly cash-flow cost rate i. [3 marks]

The net advance equals present value of all required outflows.

Answer: $4900 = 450\frac{1-(1+i)^{-12}}{i}$

(b) Solve for i and give the effective annual cost rate. [3 marks]

Use a root search on $0 \le i \le 0.1$.

Annualise with $(1+i)^{12} - 1$.

Answer: Monthly 1.5274%; annual 19.9502%.