Functions versus relations and the vertical-line test

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Decide whether $(0, 1), (1, 1), (2, 1)$ is a function of x. [2 marks]

Every input has exactly one output, although all outputs coincide.

Answer: Yes.

2. Decide whether $(-2, 0), (-2, 4), (3, 5)$ is a function of x. [2 marks]

Input -2 has two different outputs.

Answer: No.

3. Is the relation $x = 3$ a function of x? Explain. [2 marks]

The same x-value 3 permits every real y-value, so a vertical line overlaps it at many points.

Answer: No.

4. Is $y = x^2$ a function of x, and is each output produced by only one input? [2 marks]

Every x has one square.

For example, inputs 2 and -2 both produce 4.

Answer: It is a function of x, but it is not one-to-one on all real inputs.

5. Restrict $x^2 + y^2 = 16$ to give a function with non-positive outputs. [2 marks]

Select the lower semicircle, which assigns one non-positive y-value to each allowed x.

Answer: $y = -\sqrt{16 - x^2}$, $-4 \leq x \leq 4$.

6. A table lists f(0)=1, f(1)=3 and f(2)=5. Does it prove f(x)=2x+1 for every real x? [2 marks]

It agrees with that rule at the listed inputs, but many other rules have those same three values.

Answer: No.