Functions and straight-line graphs - Practice test 2

10 questions | 42 marks | 70 minutes

Use symbolic rules, intercepts and contextual restrictions. Explanations matter as well as calculations.

Equivalent correct methods earn the corresponding marks.

1. Let $f(x) = x^2 - 2x + 3$. [4 marks]

(a) Find $f(-1)$. [2 marks]

$(-1)^2-2(-1)+3=6$.

Answer: 6

(b) Simplify $f(t+1)$. [2 marks]

$(t+1)^2-2(t+1)+3=t^2+2t+1-2t-2+3=t^2+2$.

Answer: $t^2 + 2$

2. Consider the reciprocal rule $g(x) = 4/(x - 3)$ and the relation R={(2,1),(2,5),(4,3)}. [4 marks]

(a) State the domain and range of g. [2 marks]

The denominator must be non-zero.

A non-zero numerator cannot produce zero.

Answer: All real x except 3; all real outputs except 0.

(b) Explain whether R is a function of x. [2 marks]

Input 2 is paired with outputs 1 and 5.

Answer: It is not.

3. A line passes through (-2,-1) and (4,8). [4 marks]

(a) Find its equation. [2 marks]

Gradient=(8+1)/(4+2)=3/2.

Using (-2,-1), -1=-3+c gives c=2.

Answer: $y = 3x/2 + 2$

(b) Find both axis intercepts. [2 marks]

Set y=0 to get x=-4/3; set x=0 to get y=2.

Answer: (-4/3,0) and (0,2).

4. Use gradients to construct the required lines. [4 marks]

(a) Find the line perpendicular to $3x + 2y = 8$ through (3,-2). [2 marks]

The original gradient is -3/2, so the perpendicular gradient is 2/3.

Use y+2=(2/3)(x-3).

Answer: $y = 2x/3 - 4$

(b) Give the lines parallel and perpendicular to x=-5 through (2,3). [2 marks]

A line parallel to a vertical line is vertical; its perpendicular is horizontal.

Answer: Parallel: x=2. Perpendicular: y=3.

5. A collection contains 20 coins, each either a 2-dollar coin or a 50-cent coin. Their total value is 22 dollars. [5 marks]

(a) Form equations for the two numbers of coins. [2 marks]

Let a count 2-dollar coins and b count 50-cent coins.

Answer: $a + b = 20$, $2a + 0.5b = 22$.

(b) Solve and check the numbers. [3 marks]

Substitute b=20-a: 2a+10-0.5a=22, giving a=8.

Then b=12; the value is 16+6=22 dollars.

Answer: 8 two-dollar coins and 12 fifty-cent coins.

6. A segment has endpoint A(-3,4) and midpoint M(2,-1). [4 marks]

(a) Find its other endpoint B. [2 marks]

(-3+x)/2=2 gives x=7.

(4+y)/2=-1 gives y=-6.

Answer: (7,-6).

(b) Find the exact length AB. [2 marks]

The coordinate changes are 10 and -10.

$AB = \sqrt{100 + 100} = 10\sqrt{2}$.

Answer: $10\sqrt{2}$ units.

7. y is inversely proportional to x. When x=4, y=-5. [4 marks]

(a) Find the rule and evaluate y at x=-2. [2 marks]

The constant product is 4(-5)=-20.

At x=-2, y=(-20)/(-2)=10.

Answer: $y = -20/x$; y=10.

(b) State the asymptotes and explain whether y can be zero. [2 marks]

No finite permitted input makes -20/x equal zero.

Answer: Asymptotes x=0 and y=0; y is never zero.

8. The circle relation is $x^2 + (y+1)^2 = 16$. [5 marks]

(a) State its centre and radius. [2 marks]

Compare with standard circle form.

Answer: (0,-1), radius 4.

(b) Write its upper semicircle as a function and state its domain and range. [3 marks]

Choose the positive square-root branch.

The highest output is -1+4=3, and endpoint outputs are -1.

Answer: $y = -1 + \sqrt{16 - x^2}$; domain [-4,4], range [-1,3].

9. A delivery service proposes a linear fee model using the supplied table. [4 marks]

Distance d (km)	2	5	8
Fee C (dollars)	9	15	21

(a) Find the fee rule C(d). [2 marks]

A 3 km increase adds 6 dollars, so the rate is 2 dollars/km.

At d=2, 9=4+c gives c=5.

Answer: $C = 2d + 5$ dollars.

(b) Predict the fee for 10 km. State one assumption behind your prediction. [2 marks]

C(10)=2(10)+5=25.

The observed data alone do not prove that the tariff continues unchanged.

Answer: 25 dollars; the same linear tariff must apply at 10 km.

10. Two lines are $y = (k - 1)x + 2$ and $y = 3x - 5$. [4 marks]

(a) Find k for which they are parallel. Can they ever be the same line? [2 marks]

Equal slopes require k-1=3.

Their fixed y-intercepts 2 and -5 differ.

Answer: k=4; they cannot coincide.

(b) Find their intersection when k=2. [2 marks]

Then the first line is y=x+2.

x+2=3x-5 gives x=7/2, and y=11/2.

Answer: (7/2,11/2).