Functions and straight-line graphs - Practice test 3

10 questions | 43 marks | 70 minutes

Use functions to connect algebra with measurement and practical restrictions. Justify choices of models.

Equivalent correct methods earn the corresponding marks.

1. The rule is $f(x) = 3/(2 - x)$. [4 marks]

(a) Evaluate f(-1) and f(5). [2 marks]

f(-1)=3/3=1; f(5)=3/(-3)=-1.

Answer: 1 and -1.

(b) Solve f(x)=6 and state the excluded input of the original rule. [2 marks]

3=6(2-x), so 6x=9 and x=3/2.

The original denominator is zero at x=2.

Answer: $x = 3/2$; x=2 is excluded.

2. A rectangle has fixed width 4 cm and variable positive length l cm. [5 marks]

(a) Write area A as a function of l. Name the independent and dependent variables and their units. [2 marks]

Multiply length by width; area responds to the chosen length.

Answer: $A = 4l$; input l in cm, output A in cm².

(b) State suitable domains and ranges for A(l) and perimeter P(l). [3 marks]

A non-degenerate rectangle has positive length.

As l approaches zero, the perimeter approaches 8 without reaching it.

Answer: $l > 0$, $A > 0$; $P = 2l + 8$ has $P > 8$.

3. A straight line has equation $2x - 3y = 9$. [4 marks]

(a) Find its gradient and intercepts. [2 marks]

Rearrange as y=2x/3-3.

Set y=0 for the x-intercept.

Answer: Gradient 2/3; intercepts (9/2,0), (0,-3).

(b) Find the point on this line with y=5. [2 marks]

2x-15=9 gives x=12.

Answer: (12,5).

4. Line L passes through (1,2) and (5,4). [4 marks]

(a) Find its equation. [2 marks]

Gradient=(4-2)/(5-1)=1/2.

Use 2=1/2+c, giving c=3/2.

Answer: $y = x/2 + 3/2$

(b) Find the line perpendicular to L through (3,1). [2 marks]

The perpendicular slope is -2.

$y - 1 = -2(x - 3)$.

Answer: $y = -2x + 7$

5. A car park charges 8 dollars for a van and 5 dollars for a car. There are 13 vehicles and the total charge is 80 dollars. [5 marks]

(a) Form simultaneous equations for the vehicle counts. [2 marks]

v and c count vans and cars.

Answer: $v + c = 13$, $8v + 5c = 80$.

(b) Solve and interpret the solution. [3 marks]

Subtract five times the count equation from the charge equation: 3v=15.

Then v=5, c=8; their charges sum to 40+40=80 dollars.

Answer: 5 vans and 8 cars.

6. A quadrilateral has A(0,1), B(4,1), C(4,4) and D(0,4). [4 marks]

(a) Calculate the lengths of diagonals AC and BD. [2 marks]

AC has changes 4 and 3, so length 5.

BD has changes -4 and 3, so also length 5.

Answer: Both are 5 units.

(b) Calculate both diagonal midpoints. State what the matching midpoints show. [2 marks]

Average corresponding endpoint coordinates for each diagonal.

A common midpoint divides each segment into equal halves.

Answer: Both midpoints are (2,5/2), so the diagonals bisect each other.

7. A job needs 60 worker-hours. Assume all workers have the same constant work rate and can work together without interference. [4 marks]

(a) Write the completion time t in hours as a function of the positive number of workers w. Find t for 5 workers. [2 marks]

The product wt=60 under the stated model.

At w=5, t=12.

Answer: $t = 60/w$; 12 hours.

(b) Find t for 8 workers and explain one limitation of the model. [2 marks]

60/8=7.5.

The calculation assumes extra workers do not reduce anyone’s rate.

Answer: 7.5 hours; coordination or limited workspace may prevent perfect inverse scaling.

8. A circle is centred at (2,-1) and passes through (5,3). [4 marks]

(a) Find its radius and equation. [2 marks]

The radius is $\sqrt{3^2 + 4^2} = 5$.

Insert centre and radius into the standard form.

Answer: Radius 5; $(x - 2)^2 + (y + 1)^2 = 25$.

(b) Find the leftmost, rightmost, lowest and highest points. [2 marks]

Move one radius horizontally or vertically from the centre.

Answer: (-3,-1), (7,-1), (2,-6), (2,4).

9. The table describes part of a reciprocal function. [4 marks]

x	-2	-1	0	1	2
y	-4	-8	?	8	4

(a) Find its rule and complete the missing entry if possible. [2 marks]

Every listed non-zero pair has product 8.

Division by zero is undefined; do not insert 0 as the output.

Answer: $y = 8/x$; no output is defined at x=0.

(b) State whether this is a function of x and describe its two branches. [2 marks]

Each non-zero input gives one output.

Because the product is positive, input and output have matching signs.

Answer: Yes on x≠0; the branches lie in quadrants I and III.

10. An engineer proposes a linear model $y = ax + b$. It must pass through measurements (2,11) and (6,23). [5 marks]

(a) Determine a and b. [2 marks]

The gradient is (23-11)/(6-2)=3.

Use 11=3(2)+b to get b=5.

Answer: $a = 3$, $b = 5$.

(b) A third measurement is (8,32). Does the exact model pass through it? Give a careful conclusion. [3 marks]

3(8)+5=29≠32.

The exact linear rule fails to fit all three measurements. Measurement uncertainty or a different model would need investigation; two points alone do not prove a real process is linear.

Answer: No; it predicts y=29, three units lower.