Functions and straight-line graphs - Practice test 4

10 questions | 44 marks | 70 minutes

A reasoning paper with parameter restrictions, a shifted reciprocal rule and geometric modelling.

Equivalent correct methods earn the corresponding marks.

1. The linear function f satisfies f(-1)=-5 and f(3)=7. [4 marks]

(a) Find the rule f(x). [2 marks]

The slope is (7+5)/(3+1)=3.

Use -5=-3+b to obtain b=-2.

Answer: $f(x) = 3x - 2$

(b) Compare f(2t) with 2f(t). Are they equal for all t? [2 marks]

Substitute 2t into the rule for the first expression.

Double the entire output for the second; they differ by 2.

Answer: No: f(2t)=6t-2, while 2f(t)=6t-4.

2. Let $h(x) = \sqrt{10 - 2x}$. [4 marks]

(a) Find the real domain and range without a contextual restriction. [2 marks]

Require 10-2x≥0.

As x decreases without bound, the non-negative square root increases without bound.

Answer: Domain x≤5; range y≥0.

(b) Now restrict 0≤x≤5. Find the new range. [2 marks]

The rule decreases across this interval.

h(5)=0 and h(0)=sqrt(10); both endpoints are included.

Answer: $0 \le h(x) \le \sqrt{10}$.

3. Consider the line $3y - 6x = 15$. [4 marks]

(a) Find its gradient and its two intercepts. [2 marks]

Divide by 3 to obtain y=2x+5.

Setting y=0 gives x=-5/2.

Answer: Gradient 2; intercepts (-5/2,0) and (0,5).

(b) A student says y=2x+1 is perpendicular to it because the intercepts differ. Correct the statement. [2 marks]

Their gradients are both 2.

Perpendicularity depends on the gradients, not whether intercepts differ.

Answer: They are distinct parallel lines, not perpendicular.

4. For a real number p, two points are A(p,1) and B(p,6). [4 marks]

(a) State the equation and gradient of AB. [2 marks]

Both points have the same x-coordinate and distinct y-coordinates.

Answer: x=p; its gradient is undefined.

(b) Find the line perpendicular to AB through (0,-3). [2 marks]

The perpendicular to a vertical line is horizontal.

Answer: y=-3.

5. A laboratory mixes 10% and 30% saline solutions to make 20 L of a 22% solution. Assume volumes add. [5 marks]

(a) Form equations for the litres x and y of the two solutions. [2 marks]

The final salt-equivalent volume is 0.22×20=4.4 L.

Answer: $x + y = 20$, $0.10x + 0.30y = 4.4$.

(b) Solve and check the mixture. [3 marks]

Use x=20-y: 2-0.1y+0.3y=4.4 gives y=12.

Then x=8; 0.8+3.6=4.4 L of salt-equivalent content.

Answer: 8 L of 10% solution and 12 L of 30% solution.

6. Three points are A(-4,1), B(2,9), C(2,1). [4 marks]

(a) Calculate all three side lengths. [2 marks]

AC is horizontal and BC is vertical.

AB=sqrt(6²+8²)=10.

Answer: AC=6, BC=8, AB=10.

(b) Explain why the triangle is right-angled and find the midpoint of AB. [2 marks]

AC and BC are horizontal and vertical, so they are perpendicular.

The midpoint averages the endpoints of AB.

Answer: The right angle is at C; midpoint (-1,5).

7. A shifted reciprocal rule is $y = 6/(x - 2) + 1$. [5 marks]

(a) State the asymptotes, domain and range. [2 marks]

The denominator cannot be zero.

The reciprocal term cannot be zero but approaches zero for large magnitude x.

Answer: Asymptotes x=2 and y=1; domain x≠2; range y≠1.

(b) Find both axis intercepts. [3 marks]

At y=0: 6/(x-2)=-1 gives x=-4.

At x=0: y=-3+1=-2.

Answer: (-4,0) and (0,-2).

8. A circle has centre (-2,1) and passes through (1,3). [5 marks]

(a) Write its equation. [2 marks]

The squared radius is 3²+2²=13.

Answer: $(x + 2)^2 + (y - 1)^2 = 13$.

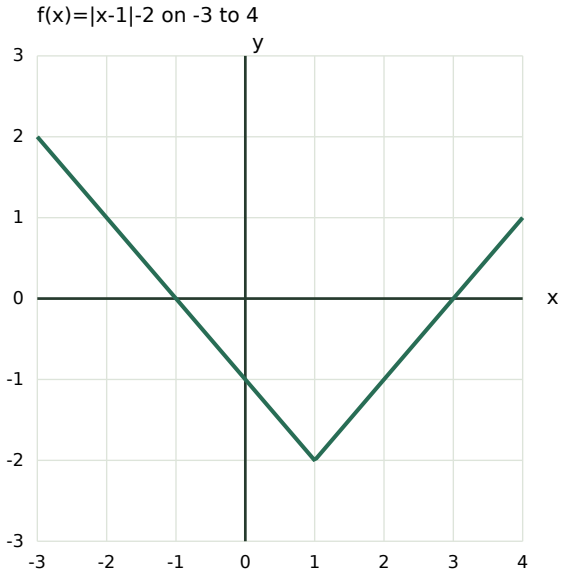
(b) Find its exact x-intercepts. [3 marks]

Set y=0: (x+2)²=12.

Take both square roots: $x + 2 = \pm 2\sqrt{3}$.

Answer: $(-2 - 2\sqrt{3}, 0)$ and $(-2 + 2\sqrt{3}, 0)$.

9. A V-shaped function is $f(x) = |x - 1| - 2$ on -3≤x≤4. Here |u| means the non-negative distance of u from zero. [4 marks]



(a) Find its minimum point and its range. [2 marks]

The absolute value is smallest at x=1.

Endpoint outputs are 2 and 1; hence the largest value on this domain is 2.

Answer: Minimum (1,-2); range [-2,2].

(b) Does its repeated output at x=0 and x=2 prevent it being a function? Explain. [2 marks]

The function definition does not require different inputs to have different outputs.

Answer: No; both outputs are -1, but each input still has one output.

10. A linear taxi tariff charges 17.50 dollars for 5 km and 27.50 dollars for 9 km. [5 marks]

(a) Find a fare rule C(d), explaining both coefficients. [2 marks]

The slope is (27.50-17.50)/(9-5)=2.50.

Using d=5 gives 17.50=12.50+b, so b=5.

Answer: $C = 2.5d + 5$; rate 2.50 dollars/km and initial fee 5 dollars.

(b) What is the greatest distance costing at most 25 dollars under this tariff? [3 marks]

2.5d+5≤25 gives d≤8.

Distance is non-negative, so the permitted distances are 0≤d≤8.

Answer: 8 km.