Functions and straight-line graphs - Practice test 5

10 questions | 50 marks | 75 minutes

A more demanding synthesis paper: justify restrictions, evaluate models and combine coordinate methods.

Equivalent correct methods earn the corresponding marks.

1. A rule is $f(x) = (x^2 - 9)/(x - 3)$, with its original algebraic domain. [5 marks]

(a) Simplify the rule, retaining the input restriction. [2 marks]

Factor x^2-9 as (x-3)(x+3).

Cancellation is permitted only if $x \neq 3$.

Answer: $f(x) = x + 3$, with $x \neq 3$.

(b) Find f(4) and explain why output 6 is missing. [3 marks]

4+3=7.

x+3=6 gives x=3, which is not in the original domain.

Answer: f(4)=7; output 6 would require the excluded input x=3.

2. A relation consists of the pairs (-1,2), (0,3) and (1,2). [4 marks]

(a) Is it a function of the first coordinate? Explain. [2 marks]

Each first coordinate occurs with exactly one output.

Answer: Yes.

(b) If input and output roles are swapped, is the reversed relation a function? Explain. [2 marks]

The reversed relation includes (2,-1) and (2,1), so input 2 has conflicting outputs.

Answer: No.

3. A line crosses the positive axes and forms a triangle of area 12 square units with them. Its y-intercept is (0,6). [4 marks]

(a) Find its positive x-intercept. [2 marks]

Using triangle area, (1/2)×b×6=12 gives b=4.

Answer: (4,0).

(b) Find the line equation and gradient. [2 marks]

Use intercepts (4,0) and (0,6).

Gradient=(0-6)/(4-0)=-3/2.

Answer: $y = -3x/2 + 6$; gradient -3/2.

4. Two lines are $y = (a + 1)x - 4$ and $y = 2x + 5$. [4 marks]

(a) Find a for which they are parallel and distinct. [2 marks]

Match slopes: a+1=2.

Their intercepts differ, so they remain distinct.

Answer: $a = 1$.

(b) Find a for which they are perpendicular. [2 marks]

The slope product must be -1: 2(a+1)=-1.

Thus a+1=-1/2 and a=-3/2.

Answer: $a = -3/2$.

5. Two pumps together supply 27 litres per minute. Pump A supplies 3 litres per minute more than twice the rate of pump B. [5 marks]

(a) Form equations for rates a and b in litres per minute. [2 marks]

Translate the combined rate and the comparison separately.

Answer: $a + b = 27$, $a - 2b = 3$.

(b) Find both rates and check them. [3 marks]

Substitute a=2b+3: 3b+3=27 gives b=8.

Then a=19; 19+8=27 and 19=2(8)+3.

Answer: A: 19 L/min; B: 8 L/min.

6. A point P(x,y) is equally distant from A(-2,0) and B(4,0). [5 marks]

(a) Write an equation using squared distances and simplify it. [3 marks]

$(x+2)^2+y^2=(x-4)^2+y^2$.

Expand and cancel x^2+y^2: $4x+4=-8x+16$, so $x=1$.

Answer: $x = 1$.

(b) Explain the geometric meaning of this result. [2 marks]

AB is horizontal, its midpoint is (1,0), and $x=1$ is the vertical line through that midpoint.

Answer: The locus is the perpendicular bisector of AB.

7. A simple gas model keeps pressure p times volume V equal to 200, with p in kPa and V in litres. Both quantities are positive. [5 marks]

(a) Write V as a function of p. Find V at p=80 and p=125. [2 marks]

Divide $pV=200$ by positive p.

$200/80=2.5$ and $200/125=1.6$.

Answer: $V = 200/p$; 2.5 L and 1.6 L.

(b) Explain why the graph is not a straight line, and state its contextual domain. [3 marks]

For example, p=50,100,150 produces V=4,2,4/3, with unequal volume changes for equal pressure changes.

Only positive pressures are meaningful under the stated model.

Answer: Its changes are not constant; $p>0$.

8. A circle has centre (3,-2) and passes through (0,2). [5 marks]

(a) Find its radius and write its equation. [2 marks]

The squared distance to the given point is $(-3)^2+4^2=25$.

Answer: Radius 5; $(x-3)^2+(y+2)^2=25$.

(b) Give the lower semicircle rule and its domain and range. [3 marks]

Choose the negative square-root branch.

The domain spans centre x-coordinate ±radius; the lower outputs run from -7 up to -2.

Answer: $y=-2-\sqrt{25-(x-3)^2}$; domain [-2,8], range [-7,-2].

9. A hire company records these charges: 8 dollars at 0 hours, 18 dollars at 2 hours and 28 dollars at 4 hours. [5 marks]

(a) Find a continuous linear model fitting the observations. [2 marks]

The gradient is (18-8)/2=5 dollars per hour; the charge at t=0 is 8 dollars.

Answer: $C(t) = 5t + 8$.

(b) The company might charge by each started hour instead. Explain why the three observations do not settle which tariff is used. [3 marks]

The data do not show what happens at a fractional hour.

A charge measured at, for example, t=0.5 h would help distinguish the proposed tariffs.

Answer: Both a continuous rate and an hourly step tariff can agree at the listed whole-hour times.

10. Two villages are at P(1,2) and Q(7,10), on axes measured in kilometres. A radio mast must be equally distant from both. [8 marks]

(a) Find the midpoint of PQ and its gradient. [2 marks]

Average coordinates to obtain (4,6).

The slope is (10-2)/(7-1)=8/6=4/3.

Answer: Midpoint (4,6); gradient 4/3.

(b) Find the perpendicular bisector of PQ. [3 marks]

Its gradient is -3/4 and it passes through (4,6).

Use y-6=(-3/4)(x-4).

Answer: $y = -3x/4 + 9$.

(c) The mast must also lie on the y-axis. Find its position and check its distance from each village. [3 marks]

Set x=0 in the bisector to get y=9.

Squared distances are $(0-1)^2+(9-2)^2=50$ and $(0-7)^2+(9-10)^2=50$.

Answer: (0,9); each distance is $\sqrt{50} = 5\sqrt{2}$ km.