Functions and straight-line graphs - Practice test 5

Name: _____ Date: _____

10 questions | 50 marks | 75 minutes

A more demanding synthesis paper: justify restrictions, evaluate models and combine coordinate methods.

A scientific calculator may be used.

1. A rule is $f(x) = (x^2 - 9)/(x - 3)$, with its original algebraic domain. [5 marks]

(a) Simplify the rule, retaining the input restriction. [2 marks]

(b) Find f(4) and explain why output 6 is missing. [3 marks]

2. A relation consists of the pairs (-1,2), (0,3) and (1,2). [4 marks]

(a) Is it a function of the first coordinate? Explain. [2 marks]

(b) If input and output roles are swapped, is the reversed relation a function? Explain. [2 marks]

3. A line crosses the positive axes and forms a triangle of area 12 square units with them. Its y-intercept is (0,6). [4 marks]

(a) Find its positive x-intercept. [2 marks]

(b) Find the line equation and gradient. [2 marks]

4. Two lines are $y = (a + 1)x - 4$ and $y = 2x + 5$. [4 marks]

(a) Find a for which they are parallel and distinct. [2 marks]

(b) Find a for which they are perpendicular. [2 marks]

5. Two pumps together supply 27 litres per minute. Pump A supplies 3 litres per minute more than twice the rate of pump B. [5 marks]

(a) Form equations for rates a and b in litres per minute. [2 marks]

(b) Find both rates and check them. [3 marks]

6. A point P(x,y) is equally distant from A(-2,0) and B(4,0). [5 marks]

(a) Write an equation using squared distances and simplify it. [3 marks]

(b) Explain the geometric meaning of this result. [2 marks]

7. A simple gas model keeps pressure p times volume V equal to 200, with p in kPa and V in litres. Both quantities are positive. [5 marks]

(a) Write V as a function of p. Find V at p=80 and p=125. [2 marks]

(b) Explain why the graph is not a straight line, and state its contextual domain. [3 marks]

8. A circle has centre (3,-2) and passes through (0,2). [5 marks]

(a) Find its radius and write its equation. [2 marks]

(b) Give the lower semicircle rule and its domain and range. [3 marks]

9. A hire company records these charges: 8 dollars at 0 hours, 18 dollars at 2 hours and 28 dollars at 4 hours. [5 marks]

(a) Find a continuous linear model fitting the observations. [2 marks]

(b) The company might charge by each started hour instead. Explain why the three observations do not settle which tariff is used. [3 marks]

10. Two villages are at P(1,2) and Q(7,10), on axes measured in kilometres. A radio mast must be equally distant from both. [8 marks]

(a) Find the midpoint of PQ and its gradient. [2 marks]

(b) Find the perpendicular bisector of PQ. [3 marks]

(c) The mast must also lie on the y-axis. Find its position and check its distance from each village. [3 marks]