Cubic and quartic graphs and models

6 questions | 15 marks

Equivalent correct methods earn the corresponding marks.

1. Describe crossing/touching roots of $y = (x + 1)(x - 2)^2$. [2 marks]

Use multiplicities one and two.

Answer: Crosses at -1; touches at 2.

2. State end behaviour of $y = -2x^4 + 3x - 1$. [2 marks]

The leading degree is even and coefficient negative.

Answer: Both ends fall.

3. Solve $x^4 - 10x^2 + 9 = 0$. [3 marks]

Let u=x squared: $(u - 1)(u - 9) = 0$.

Use both roots of each positive u.

Answer: $x = -3, -1, 1, 3$

4. Solve $x^4 + x^2 - 6 = 0$ for real x. [3 marks]

The u-equation is $(u + 3)(u - 2) = 0$.

Reject u=-3 for real x.

Answer: $x = \pm\sqrt{2}$

5. Cut squares of side x cm from the corners of a 12 cm by 8 cm sheet and fold a box. Give volume and domain. [3 marks]

The three dimensions must be positive.

Multiply height, length and width.

Answer: $V = x(12 - 2x)(8 - 2x)$ cubic centimetres; $0 < x < 4$.

6. A box has volume $V = x(12 - 2x)(8 - 2x)$ cubic centimetres. Find its volume when $x = 1$ and when $x = 2$. [2 marks]

At 1: 1(10)(6)=60.

At 2: 2(8)(4)=64.

Answer: 60 cubic centimetres and 64 cubic centimetres.