Quadratics and polynomial graphs - Practice 3

10 questions | 49 marks | 65 minutes

Tables, parameter conditions, reciprocal roots and modelling from measured data.

Equivalent correct methods earn the corresponding marks.

1. The table gives values of one quadratic function. [5 marks]

Values of y at selected x-values

x	-1	0	1	2	3
y	0	-6	-8	-6	0

(a) Use the symmetry to state the turning point and roots. [2 marks]

Equal values occur equally far from x=1.

The table shows zeros at -1 and 3.

Answer: Vertex (1,-8); roots -1 and 3.

(b) Find a rule for the function. [3 marks]

Use the two roots in factored form.

At x=0, -6=-3a, so a=2.

Answer: $y = 2(x + 1)(x - 3)$

2. Solve the two equations. [4 marks]

(a) $6x^2 - x - 2 = 0$. [2 marks]

Factor $(3x - 2)(2x + 1) = 0$.

Answer: $x = 2/3, -1/2$

(b) $x^2 - 3 = 2x$. [2 marks]

Rearrange $x^2 - 2x - 3 = (x - 3)(x + 1) = 0$.

Answer: $x = 3, -1$

3. For $f(x) = 2x^2 + 12x + k$: [5 marks]

(a) Write f in vertex form. [2 marks]

Factor 2 from the first two terms and complete the square.

Answer: $f(x) = 2(x + 3)^2 + k - 18$

(b) Find k if the minimum is 5, then explain why there are no real roots. [3 marks]

Set k-18=5.

A function with positive minimum cannot equal zero.

Answer: $k = 23$; all function values are at least 5.

4. The nonzero roots of $x^2 - 5x + 2 = 0$ are r and s. [5 marks]

(a) Find $1/r + 1/s$ and $1/(rs)$. [3 marks]

Use r+s=5 and rs=2.

$\frac{1}{r} + \frac{1}{s} = \frac{r+s}{rs}$.

Answer: 5/2 and 1/2.

(b) Form a quadratic with roots 1/r and 1/s and integer coefficients. [2 marks]

Use the new sum and product in a monic equation.

Multiply by 2.

Answer: $2x^2 - 5x + 1 = 0$

5. Consider $kx^2 - 4x + 1 = 0$. [4 marks]

(a) For nonzero k, find the condition for two distinct real roots. [3 marks]

Require discriminant 16-4k>0.

Retain the stated nonzero-leading-coefficient condition.

Answer: $k < 4$ and $k \neq 0$.

(b) Solve separately when k=0. [1 mark]

The equation becomes -4x+1=0.

Answer: $x = 1/4$

6. A family of horizontal lines y=k meets $y = (x+1)^2 + 2$. [4 marks]

(a) Give the number of intersections for k<2, k=2 and k>2. [2 marks]

The parabola has minimum 2 at x=-1.

Answer: Zero, one, and two respectively.

(b) Find the intersection coordinates when k=11. [2 marks]

Solve $(x+1)^2 = 9$.

Use both square-root branches.

Answer: (-4,11) and (2,11).

7. A monic cubic $p(x) = x^3 + ax^2 + bx + 6$ has roots 1 and -2. [6 marks]

(a) Form and solve simultaneous equations for a and b. [4 marks]

p(1)=0 gives a+b=-7.

p(-2)=0 gives 2a-b=1.

Adding gives 3a=-6.

Answer: $a = -2$, $b = -5$.

(b) Find the third root. [2 marks]

At a=-2,b=-5, divide p by $(x-1)(x+2) = x^2 + x - 2$.

The remaining factor is x-3.

Answer: 3

8. Let $g(x) = x^4 - 2x^2 - 3$. [5 marks]

(a) Find every real root exactly. [3 marks]

Set u=x squared: $(u-3)(u+1) = 0$.

Reject u=-1 for real x.

Answer: $x = \pm\sqrt{3}$

(b) Explain why the equation has fewer than four real roots. [2 marks]

A quartic can have fewer than four real roots.

The two positive/negative square-root branches of u=3 remain.

Answer: Two roots would require x squared = -1 and are not real.

9. An arch meets the ground at x=0 m and x=10 m. Its height at x=2 m is 8 m. [5 marks]

(a) Find a quadratic rule and state the physical domain. [3 marks]

Use $h = ax(x - 10)$.

Substitute (2,8) to get a=-0.5.

Answer: $h = -0.5x(x - 10)$, $0 \le x \le 10$.

(b) Find its maximum height and position. [2 marks]

The axis is halfway between the supports.

Substitute x=5.

Answer: 12.5 m at x=5 m.

10. A small event estimates attendance as $N = 120 - 4p$, where p is the ticket price in dollars and $0 < p < 30$. Revenue is pN dollars. [6 marks]

(a) Find the price giving greatest revenue and the predicted attendance. [3 marks]

Revenue $R = 120p - 4p^2 = -4(p - 15)^2 + 900$.

The vertex lies in the domain.

Answer: $p = 15$; attendance 60; revenue 900 dollars.

(b) Find both ticket prices predicted to produce 800 dollars revenue. [3 marks]

Solve $120p - 4p^2 = 800$.

Divide by -4 and factor $(p - 10)(p - 20) = 0$.

Both values lie in the allowed interval.

Answer: $p = 10$ or $p = 20$.