Trigonometry - Practice test 1

10 questions | 59 marks | 75 minutes

A balanced paper from triangle ratios to periodic applications. Give exact values where requested and retain precision in calculations.

Equivalent correct methods earn the corresponding marks.

1. A right triangle has legs 5 cm and 12 cm. Theta is opposite 5 cm. [5 marks]

(a) Find its hypotenuse and all three ratios. [3 marks]

$h = \sqrt{25 + 144} = 13$.

Apply the three ratio definitions.

Answer: $h = 13;\ \sin\theta = 5/13, \cos\theta = 12/13, \tan\theta = 5/12$.

(b) Find theta to one decimal place. [2 marks]

$\theta = \tan^{-1}(5/12) \approx 22.6199^\circ$.

Answer: 22.6°.

2. In triangle ABC, A=45°, B=75° and a=6 cm. You may use $\sin 75^\circ = (\sqrt{6} + \sqrt{2})/4$. [6 marks]

(a) Find C and b exactly. [3 marks]

$C = 180 - 45 - 75 = 60^\circ$.

$b = 6\sin 75^\circ / \sin 45^\circ = 3(\sqrt{3} + 1)$.

Answer: $C = 60^\circ;\ b = 3(\sqrt{3} + 1)$ cm.

(b) Find c exactly. [3 marks]

$c = 6\sin 60^\circ / \sin 45^\circ = 3\sqrt{6}$.

Answer: $3\sqrt{6}$ cm.

3. Given A=30°, a=10 cm and $b = 10\sqrt{3}$ cm. [6 marks]

(a) Find every possible angle B. [3 marks]

$\sin B = b\sin A/a = \sqrt{3}/2$.

Answer: 60° or 120°.

(b) Find the corresponding C values and justify the number of triangles. [3 marks]

Both choices leave a positive third angle.

Answer: C=90° or 30°; two triangles.

4. Sides 4 cm and 9 cm enclose 60°. [6 marks]

(a) Find the third side exactly. [3 marks]

$c^2 = 16 + 81 - 72(1/2) = 61$.

Answer: $\sqrt{61}$ cm

(b) Find the area exactly. [3 marks]

$K = 18\sin 60^\circ = 9\sqrt{3}$.

Answer: $9\sqrt{3}$ cm^2

5. A sector has radius 9 cm and angle 80°. [6 marks]

(a) Find its arc length and area exactly. [4 marks]

$80^\circ = 4\pi/9$; substitute in both radian formulas.

Answer: $s = 4\pi$ cm; $K = 18\pi$ cm^2.

(b) Find its full perimeter. [2 marks]

Add the two straight radii to the arc.

Answer: $18 + 4\pi$ cm

6. Work with exact trigonometric values. [6 marks]

(a) Find sine, cosine and tangent of 240°. [3 marks]

Reference angle 60° in quadrant III.

Answer: $-\sqrt{3}/2,\ -1/2,\ \sqrt{3}$

(b) Solve $\sin x = -\sqrt{2}/2$ on $0 \le x < 2\pi$. [3 marks]

Negative sine occurs in quadrantsIII andIV.

Answer: $5\pi/4, 7\pi/4$

7. Let $y = 2 + 3\cos(x/2)$. [6 marks]

(a) State the range, period and value at 0. [3 marks]

Midline 2, amplitude 3, B=1/2.

Answer: Range $[-1, 5]$; period 4π; y(0)=5.

(b) Find y=2 for $0 \le x < 4\pi$. [3 marks]

$\cos(x/2) = 0$; solve over $0 \le x/2 < 2\pi$.

Answer: $x = \pi, 3\pi$

8. Solve each equation over its stated interval. [6 marks]

(a) $2\sin x = 1,\ -\pi \le x \le \pi$. [3 marks]

Divide by 2; use positive sine.

Answer: $\pi/6, 5\pi/6$

(b) $\cos(2x) = -1,\ 0 \le x < 2\pi$. [3 marks]

2 x=π or 3π in the doubled interval.

Answer: $\pi/2, 3\pi/2$

9. A tide varies between 1.5 m and 4.5 m with period 12 h. It is at its maximum at t=0. [6 marks]

(a) Write a cosine model. [3 marks]

Midline 3, amplitude 1.5, B=π/6.

Answer: $d = 3 + 1.5\cos(\pi t/6)$

(b) Find the first two nonnegative times at mean depth. [3 marks]

Set cosine to 0 and interpret the two crossings.

Answer: 3 h and 9 h.

10. Two paths from a camp have lengths 40 m and 55 m with angle 120° between them. [6 marks]

(a) Find the distance between their endpoints, to two decimal places. [3 marks]

$d^2 = 1600 + 3025 - 4400(-1/2) = 6825$.

Answer: $5\sqrt{273}$ m ≈ 82.61 m

(b) Find the triangle's area exactly. [3 marks]

$K = \frac{1}{2}(40)(55)\sin 120^\circ$.

Answer: $550\sqrt{3}$ m^2