Trigonometry - Practice test 2

10 questions | 60 marks | 75 minutes

Use exact circular measures and explain why solutions are accepted or rejected.

Equivalent correct methods earn the corresponding marks.

1. A right triangle has hypotenuse 25 cm and one leg 7 cm. [6 marks]

(a) Find the other leg and the angle opposite 7 cm to one decimal place. [3 marks]

$a = \sqrt{625 - 49} = 24$; $\theta = \sin^{-1}(7/25) \approx 16.2602^\circ$.

Answer: 24 cm;16.3°.

(b) State the three ratios for that angle. [3 marks]

Use opposite 7, adjacent 24, hypotenuse 25.

Answer: $7/25, 24/25, 7/24$

2. A triangle has sides 9 cm and 13 cm with included angle 50°. [6 marks]

(a) Find the third side to two decimal places. [3 marks]

$c = \sqrt{81 + 169 - 234\cos 50^\circ} \approx 9.97936$.

Answer: 9.98 cm.

(b) Find its area to two decimal places. [3 marks]

$K = 58.5\sin 50^\circ \approx 44.8136$.

Answer: 44.81 cm².

3. Triangle ABC has A=25°, a=6 cm and b=9 cm. [6 marks]

(a) Find every possible B to one decimal place. [3 marks]

$\sin B = 1.5\sin 25^\circ \approx 0.633927$.

B≈39.3405° or 140.6596°.

Answer: 39.3° or 140.7°.

(b) Find corresponding C values. [3 marks]

Subtract each B and 25° from 180°.

Answer: 115.7° or 14.3°.

4. A triangle has sides 5 cm and 12 cm enclosing an obtuse angle C. Its area is 15 cm². [6 marks]

(a) Find C. [3 marks]

$15 = 30\sin C$, so sine=1/2; choose the obtuse angle.

Answer: 150°.

(b) Find the third side exactly. [3 marks]

$c^2 = 25 + 144 - 120\cos 150^\circ = 169 + 60\sqrt{3}$.

Answer: $\sqrt{169 + 60\sqrt{3}}$ cm

5. A sector has angle $5\pi/6$ and arc length 10π cm. [6 marks]

(a) Find its radius and area exactly. [4 marks]

$r = 10\pi/(5\pi/6) = 12$.

$K = 144(5\pi/6)/2 = 60\pi$.

Answer: $r = 12$ cm; $K = 60\pi$ cm^2

(b) Find the area of the complementary major sector. [2 marks]

Whole area 144π minus 60π.

Answer: 84π cm^2

6. Use the unit circle. [6 marks]

(a) Find $\cos(7\pi/6)$ and $\tan(3\pi/4)$ exactly. [3 marks]

Apply quadrant III andII signs respectively.

Answer: $-\sqrt{3}/2$ and -1

(b) Given $\sin\theta = 12/13$ in quadrant II, find cosθ and tanθ. [3 marks]

$\cos^2\theta = 1 - 144/169 = 25/169$; choose negative cosine.

Answer: $-5/13, -12/5$

7. Consider $f(x) = -1 + 2\sin(3x)$. [6 marks]

(a) State amplitude, midline and period. [3 marks]

Use |A|,D and 2π/|B|.

Answer: 2; y=−1;2π/3.

(b) Find all maxima for $0 \le x < 2\pi$. [3 marks]

$3x = \pi/2 + 2k\pi$; retain x in the interval.

Answer: $x = \pi/6, 5\pi/6, 3\pi/2$

8. Solve each equation. [6 marks]

(a) $\tan(2x) = 1$ for $0 \le x < \pi$. [3 marks]

$2x = \pi/4 + k\pi$.

Answer: $\pi/8, 5\pi/8$

(b) $2\cos^2 x - 1 = 0$ for $0 \le x < 2\pi$. [3 marks]

$\cos x = \pm\sqrt{2}/2$; use all four quadrants.

Answer: $\pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

9. A rotating arm's endpoint has height h=9+4 sin(πt/5) metres, t in seconds. [6 marks]

(a) Find its range, period and first maximum time. [3 marks]

Amplitude 4, midline 9; argumentπ/2 gives the first maximum.

Answer: [5,13] m;10 s;2.5 s.

(b) Find the first positive time when h=7. [3 marks]

$\sin(\pi t/5) = -1/2$; first positive argument 7π/6.

Answer: 35/6 s≈5.83 s.

10. A circular running track has radius 30 m. A runner travels at 3 m/s. [6 marks]

(a) How many radians does the runner turn through in 20 s? [3 marks]

Arc distance 60 m; angle 60/30=2.

Answer: 2 rad.

(b) Find the time for one full lap and explain the unit conversion. [3 marks]

Circumference 60π m; divide by 3 m/s.

Answer: 20π s ≈ 62.83 s