Trigonometry - Practice test 3

10 questions | 58 marks | 75 minutes

Connect representations and justify choices, including diagrams, tables and inverse problems.

Equivalent correct methods earn the corresponding marks.

1. A triangle has A=35°, a=12 cm and b=8 cm. [6 marks]

(a) Find B to one decimal place. [3 marks]

$\sin B = 8\sin 35^\circ/12 \approx 0.382384$.

B≈22.4814°.

Answer: 22.5°.

(b) Explain why there is only one triangle, then find C. [3 marks]

35°+157.5° exceeds 180°; C=180°−35°−B.

Answer: The supplement 157.5° is invalid; C≈122.5°.

2. Work with angle units. [5 marks]

(a) Convert 315° and−135° to radians. [2 marks]

Multiply each byπ/180 and simplify.

Answer: $7\pi/4$ and $-3\pi/4$

(b) List all angles coterminal with π/4 in $-2\pi \le x \le 2\pi$. [3 marks]

$x = \pi/4 + 2k\pi$; retain k=−1,0.

Answer: $-7\pi/4, \pi/4$

3. A sector has radius r and angle 2 radians. Its area is 25 cm². [6 marks]

(a) Find r and its arc length. [3 marks]

$25 = r^2(2)/2 = r^2$; select positive r.

Answer: r=5 cm; arc 10 cm.

(b) Find the full boundary length and compare it with the arc. [3 marks]

Boundary=10+2×5=20.

Answer: 20 cm; twice the arc.

4. A triangle has sides 10 cm,11 cm and 17 cm. [6 marks]

(a) Find its largest angle to one decimal place. [3 marks]

$\cos C = (100 + 121 - 289)/220 = -17/55$.

C≈108.00445°.

Answer: 108.0°.

(b) Explain why the result must be obtuse without calculating inverse cosine. [3 marks]

Also $17^2 > 10^2 + 11^2$; the opposite angle exceeds 90°.

Answer: The cosine is negative.

5. A chord subtends 120° at the centre of a circle of radius 6 cm. [6 marks]

(a) Find chord length exactly. [3 marks]

Bisect the isosceles triangle; half-chord=6 sin 60°.

Answer: $6\sqrt{3}$ cm

(b) Find the minor arc and explain why it exceeds the chord. [3 marks]

Arc=6×2π/3=4π≈12.57; chord≈10.39.

A curved route is longer than the straight chord.

Answer: 4π cm

6. The table records height at equal time intervals during one smooth repeating cycle. [6 marks]

Height observations during one cycle

Time t (h)	Height h (m)
0	8
2	5
4	2
6	5
8	8

(a) Give a cosine rule consistent with the smooth cycle shown by these values. [3 marks]

Midline 5, amplitude 3, full cycle 8 h and starts at maximum.

Answer: $h = 5 + 3\cos(\pi t/4)$

(b) State one assumption needed to predict h at 10 h, and give that prediction. [3 marks]

$\cos(5\pi/2) = 0$.

Answer: Assume the same cycle continues; h(10)=5 m.

7. Let $g(x) = 4 - 2\cos(2x - \pi/3)$. [6 marks]

(a) Find its amplitude, period and horizontal shift. [3 marks]

$2x - \pi/3 = 2(x - \pi/6)$.

Answer: 2;π;π/6 to the right.

(b) Solve g(x)=3 on $0 \le x < \pi$. [3 marks]

$\cos(2x - \pi/3) = 1/2$.

The allowed arguments−π/3 andπ/3 give the roots.

Answer: $x = 0, \pi/3$

8. Solve $2\sin^2 x - 3\sin x + 1 = 0$. [5 marks]

(a) Find possible sine values. [2 marks]

$(2u - 1)(u - 1) = 0$, with u=sinx.

Answer: $\sin x = 1/2$ or 1.

(b) Find every root for $0 \leq x < 2\pi$. [3 marks]

Use both positive-sine values and combine distinct roots.

Answer: $\pi/6, \pi/2, 5\pi/6$

9. Two survey rays from O have bearings 020° and 140°. Points P,Q are respectively 30 m and 50 m along the rays. [6 marks]

(a) Find the interior angle POQ and distance PQ exactly. [3 marks]

Bearing difference 120°; $PQ^2 = 900 + 2500 - 3000\cos 120^\circ = 4900$.

Answer: 120°; 70 m

(b) Find the area OPQ exactly. [3 marks]

$K = \frac{1}{2}(30)(50)\sin 120^\circ$.

Answer: $375\sqrt{3}$ m^2

10. A triangular frame uses sides 6 m and 10 m with a flexible joint. [6 marks]

(a) Find the greatest possible enclosed area and corresponding angle. [3 marks]

Area=30 sinC, whose maximum is 30.

Answer: 30 m² at 90°.

(b) Find both included angles giving area 15 m², and explain why the frames have different third sides. [3 marks]

$\sin C = 1/2$.

Cosine rule contains cosC, which changes sign for supplementary angles.

Answer: 30° and 150°; their cosines differ.