Trigonometry - Practice test 4

10 questions | 56 marks | 75 minutes

Interpret periodic models and combine triangle methods in practical situations.

Equivalent correct methods earn the corresponding marks.

1. An 8 m ramp makes 30° with horizontal ground. [5 marks]

(a) Find vertical rise and horizontal run exactly. [3 marks]

Rise=8 sin 30°; run=8 cos 30°.

Answer: 4 m and $4\sqrt{3}$ m

(b) A safety limit allows rise at most 3 m for this ramp length. Is the proposed ramp acceptable? [2 marks]

Its rise is 4 m, exceeding the limit by 1 m.

Answer: No.

2. Triangle data give A=40°, a=5 cm and b=9 cm. [5 marks]

(a) Use the sine rule to check whether any triangle exists. [3 marks]

$\sin B = 9\sin 40^\circ/5 \approx 1.1570 > 1$.

Answer: No triangle.

(b) Explain why trying the supplementary angle does not help. [2 marks]

The inconsistency occurs before inverse sine can give a real angle.

Answer: Neither angle can have sine greater than 1.

3. A triangle has sides 7 cm,8 cm and 9 cm. [6 marks]

(a) Find the cosine of the angle C opposite 9 cm. [3 marks]

$\cos C = (49 + 64 - 81)/(2 \cdot 7 \cdot 8) = 2/7.$

Answer: 2/7

(b) Use this to find its exact area. [3 marks]

$\sin C = \sqrt{1 - 4/49} = 3\sqrt{5}/7.$

$K = 28(3\sqrt{5}/7) = 12\sqrt{5}.$

Answer: $12\sqrt{5}$ cm^2

4. A sector has angle 3 radians and full perimeter 10 cm. [6 marks]

(a) Find radius and arc length. [3 marks]

P=rθ+2 r=5 r=10.

Answer: r=2 cm; arc 6 cm.

(b) Find its area and describe why 3 radians is a valid sector angle. [3 marks]

$K = \frac{1}{2}(4)(3) = 6.$

Answer: 6 cm²; it is between 0 and 2π.

5. An angle θ is in quadrant IV and $\cos\theta = 5/13$. [5 marks]

(a) Find sine and tangent exactly. [3 marks]

$\sin^2\theta = 144/169$; choose negative sine.

Answer: $-12/13, -12/5$

(b) Find $\sin^2\theta + \cos^2\theta$ and explain its geometric meaning. [2 marks]

$144/169 + 25/169 = 1.$

Answer: 1; squared coordinates sum to the squared unit radius.

6. A height model is $h = 6 - 4\cos(\pi t/6)$ m, with t in seconds. [6 marks]

(a) Find range, period and starting value. [3 marks]

Read midline/amplitude; evaluate t=0.

Answer: [2,10] m;12 s;2 m.

(b) Find h=8 during the first cycle 0≤t<12. [3 marks]

$\cos(\pi t/6) = -1/2$; arguments 2π/3 and 4π/3.

Answer: t=4 s and 8 s.

7. A student claims sin(2 x)=2 sinx for all x. [5 marks]

(a) Give an exact counterexample. [3 marks]

$\sin \pi = 0$, whereas $2\sin(\pi/2) = 2$.

Answer: x=π/2: left 0, right 2.

(b) Explain the difference between an equation and an identity. [2 marks]

One counterexample disproves a universal identity.

Answer: An equation can hold for selected x; an identity holds throughout its domain.

8. Consider $y = \tan(4x)$. [6 marks]

(a) State its period and closest asymptotes to 0. [3 marks]

$T = \pi/4$; solve 4 x=±π/2.

Answer: π/4; x=±π/8.

(b) Solve y=0 on $-\pi/4 \le x \le \pi/4$. [3 marks]

4 x is an integer multiple ofπ.

Answer: $x = -\pi/4, 0, \pi/4$

9. A wheel has centre 15 m above ground, radius 12 m and period 48 s. A rider starts at the highest point. [6 marks]

(a) Write a height rule. [3 marks]

Midline 15; amplitude 12; B=2π/48.

Answer: $h = 15 + 12\cos(\pi t/24)$

(b) Find the first two times h=9 m. [3 marks]

$\cos(\pi t/24) = -1/2$; arguments 2π/3 and 4π/3.

Answer: 16 s and 32 s.

10. Two observers are 20 m apart on level ground on the same side of a tower. The nearer angle of elevation is 60° and the farther is 30°. [6 marks]

(a) Find the nearer observer's distance from the base. [3 marks]

Let it be x. Then $h = x\sqrt{3} = (x + 20)/\sqrt{3}$.

3 x=x+20, so x=10.

Answer: 10 m.

(b) Find tower height exactly and state one modelling assumption. [3 marks]

$h = 10\tan 60^\circ = 10\sqrt{3}$.

The tower is vertical and the base/observer reference levels match.

Answer: $10\sqrt{3}$ m; observers at the same ground/eye reference level.