Trigonometry - Applications and reasoning test

10 questions | 58 marks | 75 minutes

A more demanding paper requiring method selection, domain checks and interpretation. All necessary data are supplied.

Equivalent correct methods earn the corresponding marks.

1. A triangle is claimed to have A=60°, a=4 cm and b=8 cm. [6 marks]

(a) Show whether the data are possible. [3 marks]

$\sin B = 8\sin 60^\circ/4 = \sqrt{3} > 1$.

Answer: Impossible.

(b) If only b changes, find the greatest b compatible with the sine bound. State whether it gives a valid triangle. [3 marks]

$b\sin 60^\circ/4 \le 1$.

At equality B=90°, leaving positive C=30°.

Answer: $b = 8/\sqrt{3} = 8\sqrt{3}/3$ cm; valid with B=90°, C=30°.

2. Sides 4 cm and 6 cm enclose 150°. [6 marks]

(a) Find the third side and area exactly. [4 marks]

$c^2 = 16 + 36 - 48(-\sqrt{3}/2)$.

$K = 12\sin 150^\circ = 6$.

Answer: $c = \sqrt{52 + 24\sqrt{3}}$ cm; $K = 6$ cm^2

(b) An included angle 30° gives the same area. Does it give the same third side? Explain. [2 marks]

Supplementary angles have equal sine but opposite cosine.

Answer: No; its squared third side is $52-24\sqrt{3}$.

3. A triangle has A=45°, a=7 cm and b=8 cm. [6 marks]

(a) Find all possible B values to one decimal place. [3 marks]

$\sin B = 8\sin 45^\circ/7 = 4\sqrt{2}/7$.

B≈53.9129° or 126.0871°.

Answer: 53.9° or 126.1°.

(b) Find corresponding third angles and decide how many triangles exist. [3 marks]

C=135°−B; both are positive.

Answer: 81.1° or 8.9°; two triangles.

4. Two sectors each have full perimeter 20 cm. Sector I has angle 1 radian; sectorII has angle 2 radians. [6 marks]

(a) Find each radius and exact area. [4 marks]

Use 20=r(θ+2).

Substitute each radius into$K=r^2\theta/2$.

Answer: I: $r = 20/3$, $K = 200/9$; II: r=5, K=25.

(b) Which has greater area and by how much? [2 marks]

$25 - 200/9 = 25/9$.

Answer: Sector II by $25/9$ cm^2.

5. A periodic displacement has minimum 2 cm, maximum 14 cm and period 10 s. At t=0 it crosses the midline downwards. [6 marks]

(a) Write a sine rule consistent with the description. [3 marks]

Midline 8, amplitude 6, B=π/5; negative sine initially falls.

Answer: $s = 8 - 6\sin(\pi t/5)$

(b) Find the first minimum and first return to the same downward crossing. [3 marks]

The first sine maximum occurs at argumentπ/2; one full cycle returns after 10 s.

Answer: Minimum at 2.5 s; downward crossing at 10 s.

6. Solve $\sin^2 x + \sin x = 0$. [5 marks]

(a) Find possible sine values. [2 marks]

$\sin x(\sin x + 1) = 0$.

Answer: 0 or -1.

(b) Find all roots on $0 \le x < 2\pi$. [3 marks]

Combine sinx=0 with sinx=−1; exclude 2π.

Answer: $x = 0, \pi, 3\pi/2$

7. Consider $\cos(3x - \pi/2) = 0$ for $0 \le x \le \pi$. [5 marks]

(a) Transform the x interval into an interval for the argument. [2 marks]

Multiply by 3, then subtractπ/2.

Answer: $-\pi/2 \le 3x - \pi/2 \le 5\pi/2$

(b) Find every solution. [3 marks]

Argument zeros are−π/2,π/2,3π/2,5π/2; solve for x.

Answer: $0, \pi/3, 2\pi/3, \pi$

8. An isosceles triangular panel has two equal sides 10 cm and included angle C. [6 marks]

(a) Find C values giving area 25 cm². [3 marks]

$25 = 50\sin C$, so sine=1/2.

Answer: 30° or 150°.

(b) Find the base length for each panel exactly and compare the shapes. [3 marks]

$b^2 = 200 - 200\cos C$.

Use cos 30°=√3/2 and cos 150°=−√3/2.

Answer: $\sqrt{200 - 100\sqrt{3}}$ cm or $\sqrt{200 + 100\sqrt{3}}$ cm; the obtuse panel has the longer base.

9. A proposed cosine model is h=5+2 cos(πt/4). Measurements at t=0,2,4 h are respectively 7,5,4 m. [6 marks]

(a) Identify which measurement disagrees with the model and give the predicted value. [3 marks]

$\cos \pi = -1$, so h(4)=3.

Answer: The 4 h measurement; the model predicts 3 m.

(b) Explain two possible responses before using the model for a long prediction. [3 marks]

Do not discard a conflicting measurement merely to protect a chosen model.

Obtain more observations to decide whether the model is suitable.

Answer: Check measurement reliability and reconsider the fitted parameters or periodic assumptions.

10. A walker goes 100 m east from A to B, then 60 m on bearing 330° to C. [6 marks]

(a) Find C's east and north displacements from A exactly. [3 marks]

The second leg has east component 60 sin 330°=−30 and north component 60 cos 330°=30√3.

Answer: East 70 m; north $30\sqrt{3}$ m.

(b) Find AC exactly and the bearing of C from A to one decimal place. [3 marks]

$AC = \sqrt{70^2 + (30\sqrt{3})^2} = 20\sqrt{19}$.

Bearing $= \tan^{-1}(70/(30\sqrt{3})) \approx 53.4132^\circ$.

Answer: $20\sqrt{19}$ m ≈ 87.18 m; bearing 053.4°.