Logarithm laws

6 questions | 15 marks

Equivalent correct methods earn the corresponding marks.

1. Combine $\log a + \log b$ for a,b>0. [2 marks]

A product inside one log equals the sum of the logs.

Answer: $\log(ab)$

2. Expand $\log(100x^2/y)$ for x,y>0. [2 marks]

$\log 100 = 2$; apply quotient and power laws.

Answer: $2 + 2\log x - \log y$

3. Combine $2\log x - \log y$ for x,y>0. [2 marks]

Move 2 into the exponent, then use the quotient law.

Answer: $\log(x^2/y)$

4. Solve $\log(x+4) - \log x = \log 3$. [3 marks]

Domain x>0; $(x+4)/x = 3$, giving x+4=3x and x=2.

Answer: x=2

5. Solve $\log x + \log(x-2) = \log 8$. [3 marks]

Domain x>2; $x(x-2) = 8$, so $(x-4)(x+2) = 0$.

Reject -2; retain 4.

Answer: x=4

6. Compare $\log(1+9)$ with $\log 1 + \log 9$. Explain what this tests. [3 marks]

$\log 10 = 1$ but $\log 1 + \log 9 = 0 + \log 9$.

Answer: The first is 1; the second is approximately 0.95424; the proposed sum-splitting rule fails.