Marking guide | Exponentials and logarithms

Exponential and logarithmic functions - Practice test 2

10 questions | 46 marks | 70 minutes

Interpret tables, transformed curves and inverse relationships before analysing two competing models.

Equivalent correct methods earn the corresponding marks.

1. Use exact forms before decimal approximations. [4 marks]

(a) Evaluate $64^{2/3}$ and $81^{-3/4}$. [2 marks]

$\sqrt[3]{64} = 4$, then square; $\sqrt[4]{81} = 3$, then cube and invert.

Answer: 16 and 1/27.

(b) Simplify $\sqrt{48} + 2\sqrt{27}$. [2 marks]

$4\sqrt{3} + 6\sqrt{3} = 10\sqrt{3}$.

Answer: $10\sqrt{3}$

2. Decide whether each claim is valid and justify it. [4 marks]

(a) $\sqrt{x^2} = x$ for every real x. [2 marks]

For x=-3, the left side is 3 and the right side is -3.

Answer: False; the left side is the absolute value of x.

(b) $\log(2x) = \log 2 + \log x$ for x>0. [2 marks]

It is the product law with positive arguments and matching bases.

Answer: True.

3. Use the equally spaced observations in the table. [4 marks]

Daily observations

t (days)	0	1	2
Amount	160	120	90

(a) Find a suitable exponential rule. State the daily percentage change. [2 marks]

120/160=90/120=0.75; retain 75% each day.

Answer: $A(t) = 160(0.75)^t$; 25% daily decrease.

(b) Find the predicted amount at t=4. [2 marks]

$160(0.75)^4 = 50.625$.

Answer: 50.625

4. Consider $g(x) = 7 - 3(2^{x+1})$. [6 marks]

(a) Find its y-intercept, asymptote and range. [3 marks]

$g(0) = 7 - 3(2) = 1$. The subtracted exponential is positive.

Answer: $(0, 1)$; y=7; range $y < 7$.

(b) Find its x-intercept to three decimals. [3 marks]

$2^{x+1} = 7/3$, so $x = \log(7/3)/\log 2 - 1$.

Answer: (0.222,0)

5. An exponential model passes through (2,45) and (5,151.875). [4 marks]

(a) Find its one-unit multiplier. [2 marks]

$b^3 = 151.875/45 = 3.375$, so b=1.5.

Answer: 1.5

(b) Find the initial value and the rule. [2 marks]

$A = 45/(1.5)^2 = 20$.

Answer: $A = 20$; $y = 20(1.5)^t$.

6. A 320 mg sample has a 6-hour half-life. [5 marks]

(a) Write a model with t in hours and find its mass after 18 hours. [2 marks]

18 hours is three half-lives; 320/8=40.

Answer: $M(t) = 320(1/2)^{t/6}$; 40 mg.

(b) Find the time when 100 mg remains, to two decimals. [3 marks]

$t = 6\log(100/320)/\log(1/2)$.

Answer: 10.07 hours.

7. Work with logarithmic statements. [4 marks]

(a) Write $\log_5 z = -2$ in exponential form and find z. [2 marks]

The log value is the exponent on base 5.

Answer: $z = 5^{-2} = 1/25$

(b) Find the domain of $\log(8 - 2x)$. [2 marks]

8-2x must be positive.

Answer: $x < 4$

8. Use log laws and check restrictions. [5 marks]

(a) Combine $3\log a + \log b - \log c$ for positive a,b,c. [2 marks]

Apply power, product and quotient laws.

Answer: $\log(a^3 b/c)$

(b) Solve $\log(x+6) - \log x = \log 4$. [3 marks]

Domain x>0; $(x+6)/x = 4$, giving x=2.

Answer: x=2.

9. An amount obeys $Q(t) = 90(0.92)^t$. [5 marks]

(a) Solve $Q(t) < 30$ for real t. [3 marks]

90(0.92)^t<30 gives $(0.92)^t < 1/3$; dividing by a negative log reverses the inequality.

Answer: $t > \log(1/3)/\log 0.92 \approx 13.176$

(b) If measured yearly, state the first completed year satisfying it. [2 marks]

Choose the first integer strictly greater than the real crossing time.

Answer: 14

10. Two proposed sales models are $L(t) = 100 + 20t$ and $E(t) = 100(1.15)^t$, with t in months. [5 marks]

(a) Compare their predictions after two months. [2 marks]

$L(2) = 140$ and $E(2) = 100(1.15)^2 = 132.25$.

Answer: L=140; E=132.25.

(b) Explain what extra evidence is needed to choose between the models. [3 marks]

Two plausible rules do not establish whether changes are additive or multiplicative.

Compare additional observations and consider capacity/seasonal effects.

Answer: Further time-matched observations and a plausible description of how sales change.