Exponential and logarithmic functions - Practice test 4

10 questions | 48 marks | 75 minutes

Test the reasoning behind algebraic rules, domains, graph features and model selection.

Equivalent correct methods earn the corresponding marks.

1. Consider the definition of a real logarithm. [4 marks]

(a) State the conditions on base b and argument x for log base b of x. [2 marks]

A valid real exponential base must be positive and non-unit; its outputs are positive.

Answer: b>0, b not equal to 1, and x>0.

(b) Use the definition to evaluate $\log_{1/3} 9$. [2 marks]

$(1/3)^{-2} = 9$.

Answer: -2

2. An asset gains 20% in one year and then loses 20% the following year. [5 marks]

(a) Find the combined multiplier and overall percentage change. [2 marks]

$1.2(0.8) = 0.96$.

Answer: 0.96; 4% decrease.

(b) After a 20% loss, what percentage increase restores the original value? [3 marks]

The required restoring factor is $1/0.8 = 1.25$.

Answer: 25%.

3. Disprove each statement with an explicit numerical counterexample. [4 marks]

(a) $\log(a+b) = \log a + \log b$ for all positive a,b. [2 marks]

$\log(1+1) = \log 2 \neq 0 = \log 1 + \log 1$.

Answer: For a=b=1, the left side is log 2, while the right side is 0.

(b) $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ for all non-negative a,b. [2 marks]

$\sqrt{25} = 5$ but $3 + 4 = 7$.

Answer: For a=9,b=16, the sides are 5 and 7.

4. Consider $f(x) = 12 - 3(4^{x-2})$. [5 marks]

(a) Find its y-intercept, horizontal asymptote and range. [3 marks]

$f(0) = 12 - 3/16 = 11.8125$; the subtracted term is always positive.

Answer: (0,11.8125); y=12; outputs less than 12.

(b) Find the exact x-intercept. [2 marks]

$4^{x-2} = 4$, so x-2=1 and x=3.

Answer: (3,0)

5. Solve $2^{2x} - 5(2^x) + 4 = 0$. [5 marks]

(a) Use a substitution to obtain a quadratic. [2 marks]

$2^{2x} = (2^x)^2$.

Answer: Let $u = 2^x > 0$; then $u^2 - 5u + 4 = 0$.

(b) Find all real x. [3 marks]

$(u-1)(u-4) = 0$ gives u=1 or 4.

Both are positive; solve 2 to the power x equals 1 or 4.

Answer: x=0 or x=2.

6. A positive exponential model passes through (1,12) and (3,108). [5 marks]

(a) Find its rule. [3 marks]

$b^2 = 108/12 = 9$, so b=3; A=12/3=4.

Answer: $y = 4(3^t)$

(b) Explain why the negative square root for b is not used. [2 marks]

A negative base would not define a real-valued exponential function for every real t.

Answer: The model uses a positive real exponential base.

7. Compare equations that appear similar. [5 marks]

(a) Solve $\log(x^2) = 2$. [2 marks]

Original domain is x not zero; $x^2 = 100$ admits both signs.

Answer: x=10 or x=-10.

(b) Solve $2\log x = 2$ and explain the difference. [3 marks]

$\log x = 1$ implies x=10.

The identity with 2 log x assumes positive x.

Answer: x=10 only; its original domain is x>0.

8. Solve $\log_{0.5} x > 3$. [5 marks]

(a) State the domain and solve the inequality. [3 marks]

The base lies between zero and one, so its logarithm is decreasing.

$x < (0.5)^3 = 1/8$, with x>0.

Answer: $0 < x < 1/8$

(b) Check one value inside and one outside your solution interval. [2 marks]

The logs are 4 and 2, respectively.

Answer: For example x=1/16 works; x=1/4 does not.

9. A store fits exponential demand using observations D(1)=240 and D(2)=288. [4 marks]

(a) Find D(0) under the fitted rule. [2 marks]

The multiplier is 288/240=1.2; divide 240 by 1.2 to return to time zero.

Answer: 200

(b) A student uses 240 as the initial value in D(t)=240 times 1.2 to the power t. Explain the error. [2 marks]

Its prediction at t=1 is 288 rather than the stated 240.

Answer: It treats the time-one observation as time zero.

10. A school wants a model for the number of students in a room over a lesson. Arrivals stop when the room is full. [6 marks]

(a) Explain why observing rapid initial growth is not enough to justify unbounded exponential growth. [3 marks]

A constant multiplier cannot keep applying after arrivals stop at capacity.

Answer: A finite room capacity changes the future process.

(b) Describe two checks you would make before using an exponential forecast. [3 marks]

Use further observations, not only two points.

Confirm that the assumed multiplier remains plausible before the capacity limit.

Answer: Check ratios over equal time intervals and check the time range/physical assumptions.