Exponential and logarithmic functions - Applications test

Name: _____ Date: _____

10 questions | 53 marks | 75 minutes

Apply exact algebra and logarithmic models to medicine, finance, calibrated scales and the reliability of forecasts.

A scientific calculator may be used.

1. Calculate exact values and handle signs carefully. [4 marks]

(a) Evaluate $125^{2/3}$ and $16^{-1/2}$. [2 marks]

(b) Simplify $\sqrt{x^2}$ for x=-7 and explain why it is not x. [2 marks]

2. Work exactly before rounding. [4 marks]

(a) Rationalise $6/(\sqrt{7}-1)$. [2 marks]

(b) Find its value to three decimal places. [2 marks]

3. Two laboratory quantities, in milligrams, follow $A(t)=120(1.25)^t$ and $B(t)=300(0.9)^t$, where t is hours. Assume both rules are valid for $0 \le t \le 5$. [5 marks]

(a) Form an equation for the time when the amounts are equal. [2 marks]

(b) Solve the equation and identify which amount is larger at t=0. [3 marks]

4. A medicine model is M(t)=48 times 0.5 to the power t/4 mg, where t is hours after one dose. Assume no further doses. [5 marks]

(a) State its half-life and calculate M(10). [2 marks]

(b) Find all times $t \geq 0$ for which the model predicts less than 5 mg. [3 marks]

5. A transformed exponential has horizontal asymptote y=2 and passes through (0,5) and (1,11). Its rule is y=k+A times b to the power x. [5 marks]

(a) Find k, A and b. [3 marks]

(b) Predict its value at x=2 and state its range. [2 marks]

6. A scale is defined by $p = -\log C$, where C is a positive concentration in a fixed unit. [6 marks]

(a) Find C if p=4.5. [3 marks]

(b) If p increases from 4.5 to 6.5, by what factor does C change? [3 marks]

7. A sound-level model is $L = 10\log(I/I_0)$, where I and the fixed reference I_0 are positive intensities. [5 marks]

(a) Find the increase in L when I is multiplied by 100. [2 marks]

(b) A level rises by 3 units. Find the intensity multiplier to three decimals. [3 marks]

8. An initial 6000 dollars earns a nominal 4.8% per annum compounded monthly, with no fees or extra payments. [6 marks]

(a) Write the balance after m complete months and calculate it after 18 months. [3 marks]

(b) Find the first complete month in which the balance exceeds 7000 dollars. [3 marks]

9. Solve $2\log(x-1)=\log(3x+1)$. [5 marks]

(a) State the original domain and obtain a polynomial equation. [3 marks]

(b) Find the valid root and verify it. [2 marks]

10. A researcher fits the first three equally spaced observations below with a pure exponential model. [8 marks]

Observed quantity

Time t	0	1	2	3
Observed value	50	75	112.5	160

(a) Find that model and its prediction at t=3. [3 marks]

(b) The actual t=3 observation is 160. Find the absolute error and the percentage error relative to the actual value. [3 marks]

(c) Explain why this fourth observation matters before forecasting much further. [2 marks]