Stationary points

6 questions | 21 marks

Equivalent correct methods earn the corresponding marks.

1. Find and classify the stationary point of $f(x) = x^2 + 2x + 4$. [3 marks]

$f'(x) = 2x + 2 = 0$ at -1.

$f(-1) = 3$; signs change $-$ to $+$.

Answer: Minimum $(-1,3)$

2. Classify the stationary point of $g(x) = -x^2 + 6x - 1$. [3 marks]

$g'(x) = -2x + 6 = 0$ at 3.

$g(3) = 8$; signs change $+$ to $-$.

Answer: Maximum (3,8)

3. Describe the stationary point of $h(x) = x^3 + 2$. [3 marks]

$h'(x) = 3x^2$ is zero at 0.

The derivative is positive on both sides, so there is no turn.

Answer: (0,2), neither maximum nor minimum.

4. Classify all stationary points of $p(x) = x^3 - 12x$. [4 marks]

$p'(x) = 3(x - 2)(x + 2)$.

Signs $+$, $-$, $+$ give the types; evaluate p at -2 and 2.

Answer: Maximum $(-2,16)$; minimum $(2,-16)$.

5. Classify all stationary points of $q(x) = x^4 - 8x^2$. [5 marks]

$q'(x) = 4x(x - 2)(x + 2)$.

Signs across -2, 0 and 2 are $-$, $+$, $-$, $+$.

Answer: Maximum (0,0); minima $(-2,-16)$ and $(2,-16)$.

6. Explain why $f(x) = x^4 + 1$ has a stationary minimum at 0. [3 marks]

$f'(x) = 4x^3$ is negative before 0 and positive after 0.

Answer: The point is (0,1); the derivative changes $-$ to $+$.