First-derivative graph analysis

6 questions | 24 marks

Equivalent correct methods earn the corresponding marks.

1. List intercepts, stationary point and monotonic intervals of $f(x) = x^2 - 4x + 3$. [5 marks]

$f = (x - 1)(x - 3)$; $f' = 2x - 4$.

Evaluate f at 0 and 2 and use derivative signs.

Answer: Roots 1,3; y-intercept 3; minimum (2,−1); decreasing x<2, increasing x>2.

2. Analyse $f(x) = x^3 - 3x^2$. [5 marks]

$f = x^2(x - 3)$ and $f' = 3x(x - 2)$.

The derivative signs are +, −, +.

Answer: Intercepts (0,0),(3,0); maximum (0,0); minimum (2,−4).

3. A differentiable function has derivative zero at x=−2 and x=1. Its derivative is positive for x<−2, negative for −2<x<1, and positive for x>1. State the stationary types. [3 marks]

The signs change + to − at −2 and − to + at 1.

No heights can be found from this information alone.

Answer: Maximum at x=−2; minimum at x=1.

4. Find roots and stationary points of $g(x) = x^3 - 3x$. [5 marks]

$g = x(x^2 - 3)$ and $g' = 3(x - 1)(x + 1)$.

Use signs +, −, + and evaluate g at ±1.

Answer: Roots $0, \pm\sqrt{3}$; maximum $(-1, 2)$, minimum $(1, -2)$.

5. A straight-line function has $f'(x) = 2$ and $f(0) = -3$. Find f. [3 marks]

A line of gradient 2 has form 2x+c.

The intercept condition makes c=−3.

Answer: $f(x) = 2x - 3$

6. Why can the derivative alone not determine vertical position? [3 marks]

For example x^2 and x^2+5 both have derivative 2x but different heights.

Answer: Adding a constant leaves the derivative unchanged.