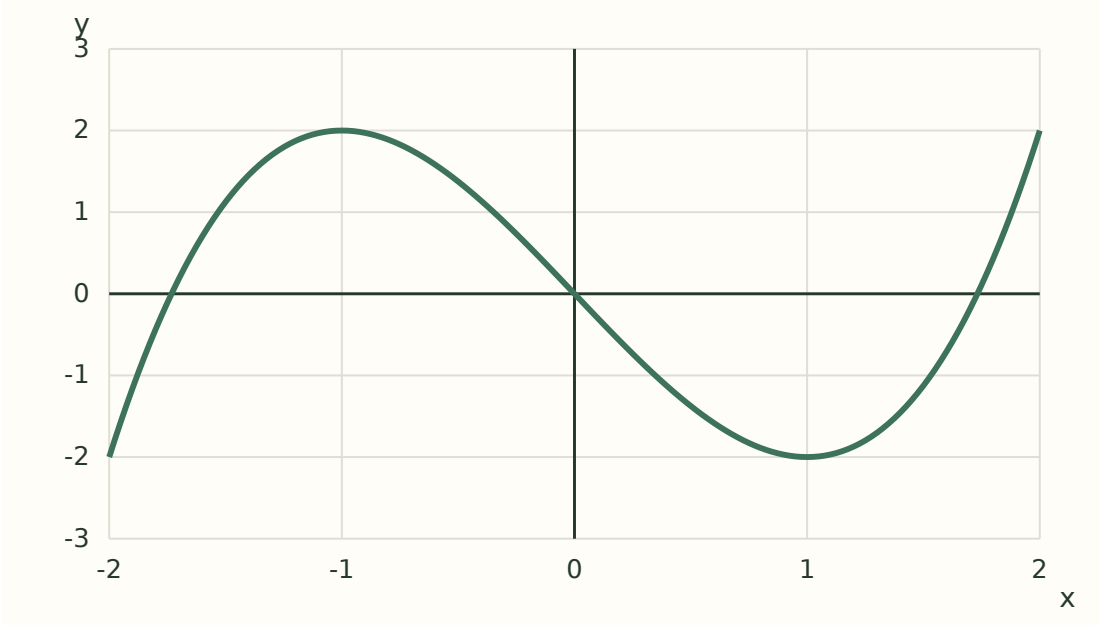
Introduction to calculus - Topic test 4

10 questions | 49 marks | 65 minutes

Analyse different representations, tangent conditions, changes of direction and the validity of solutions.

Equivalent correct methods earn the corresponding marks.

1. The supplied graph represents $f(x) = x^3 - 3x$. [5 marks]



(a) Read the approximate stationary coordinates from the graph. [2 marks]

Identify the horizontal turning points shown.

Answer: Approximately $(-1,2)$ and $(1,-2)$.

(b) Verify them exactly with a derivative. [3 marks]

$f' = 3x^2 - 3$ is zero at ± 1; signs are $+,-,+$.

Substitute into f.

Answer: Maximum $(-1,2)$, minimum $(1,-2)$.

2. Compare $f(x) = x^2$ and $g(x) = 2x^2 + 3$. [4 marks]

(a) Find both derivative functions. [2 marks]

Differentiate each expression term by term.

Answer: $f' = 2x$, $g' = 4x$.

(b) Explain how the factor 2 and the constant 3 affect gradients. [2 marks]

For every x, g′(x)=2f′(x). The derivative of 3 is 0.

Answer: The factor doubles gradients; the vertical shift does not affect them.

3. $f(x) = x^3 + kx$. Its tangent at x=1 has gradient 5. [4 marks]

(a) Find k. [2 marks]

$f' = 3x^2 + k$; $3 + k = 5$.

Answer: 2

(b) Find the tangent equation at that point. [2 marks]

With k=2, f(1)=3; y−3=5(x−1).

Answer: $y = 5x - 2$

4. For $f(x) = x^3 - x$ at the origin, [4 marks]

(a) find the tangent equation. [2 marks]

$f' = 3x^2 - 1$ gives slope -1 at 0.

Answer: $y = -x$

(b) find the normal and verify perpendicularity. [2 marks]

The normal gradient is $-1/(-1) = 1$.

Both pass through the origin and their gradients multiply to -1.

Answer: $y = x$; slope product -1.

5. Analyse $f(x) = x(x-2)^2$. [6 marks]

(a) Find intercepts and stationary coordinates. [4 marks]

$f = x^3 - 4x^2 + 4x$; $f' = (3x-2)(x-2)$.

$f(2/3) = 32/27$ and f(2)=0.

Answer: Intercepts x=0,2; stationary (2/3,32/27) and (2,0).

(b) Classify the stationary points. [2 marks]

Derivative signs are +,−,+.

Answer: Maximum at 2/3; minimum at 2.

6. $s(t) = t^3 - 9t^2 + 24t$ metres for $0 \le t \le 6$. [6 marks]

(a) Find rest times and total distance. [4 marks]

$v = 3(t-2)(t-4)$; positions at 0,2,4,6 are 0,20,16,36.

Distance=20+4+20=44.

Answer: Rest at 2s and 4s; distance 44m.

(b) At t=3, find acceleration and decide whether the particle is at rest. [2 marks]

$a = 6t - 18$ gives 0; v(3)=27−54+24=−3.

Answer: a=0 but v=−3m/s, so not at rest.

7. For $C(x) = x^2 + 2x + 3$, [4 marks]

(a) find its unrestricted minimum. [2 marks]

$C' = 2x + 2 = 0$ gives -1; C(−1)=2.

Answer: 2 at x=−1.

(b) find its minimum if only 0≤x≤4 is allowed. [2 marks]

The function increases throughout [0,4]; evaluate the left endpoint.

Answer: 3 at x=0.

8. A rectangular enclosure has a single internal divider parallel to its width. Total fencing is 60m. Let width be x and length y. [6 marks]

(a) Form the area in terms of x alone. [3 marks]

There are three widths and two lengths: 3x+2y=60.

Hence y=30−1.5x and A=xy.

Answer: $A = 30x - 1.5x^2$, with 0<x<20.

(b) Find the greatest area and dimensions. [3 marks]

$A' = 30 - 3x = 0$ gives 10; signs + then −.

Then y=15 and A=150.

Answer: 150m²; width 10m and length 15m.

9. $s(t) = t^3 - 6t^2 + 12t$ metres on $0 \le t \le 4$ seconds. [5 marks]

(a) Show the particle is at rest once inside the interval, without turning. [3 marks]

$v = 3(t-2)^2$ is zero only at 2 and positive elsewhere.

Answer: At t=2 only; velocity is nonnegative on both sides.

(b) Find displacement and total distance. [2 marks]

Positions are s(0)=0 and s(4)=16. With no reverse motion, distance equals displacement.

Answer: Both 16m.

10. A student uses $A = x(20 - x)$ for a rectangle and writes $A' = 20 - x$, obtaining x=20. [5 marks]

(a) Correct the derivative and the stationary input. [2 marks]

Expand $A = 20x - x^2$ before differentiating.

Answer: $A' = 20 - 2x$, x=10.

(b) Explain the geometric fault in the proposed x=20 answer. [3 marks]

The physical domain is 0<x<20, and the correct maximum is A(10)=100.

Answer: It gives the other side 0 and area 0; it is not a rectangle of maximum area.