# Introduction to calculus - Topic test 4

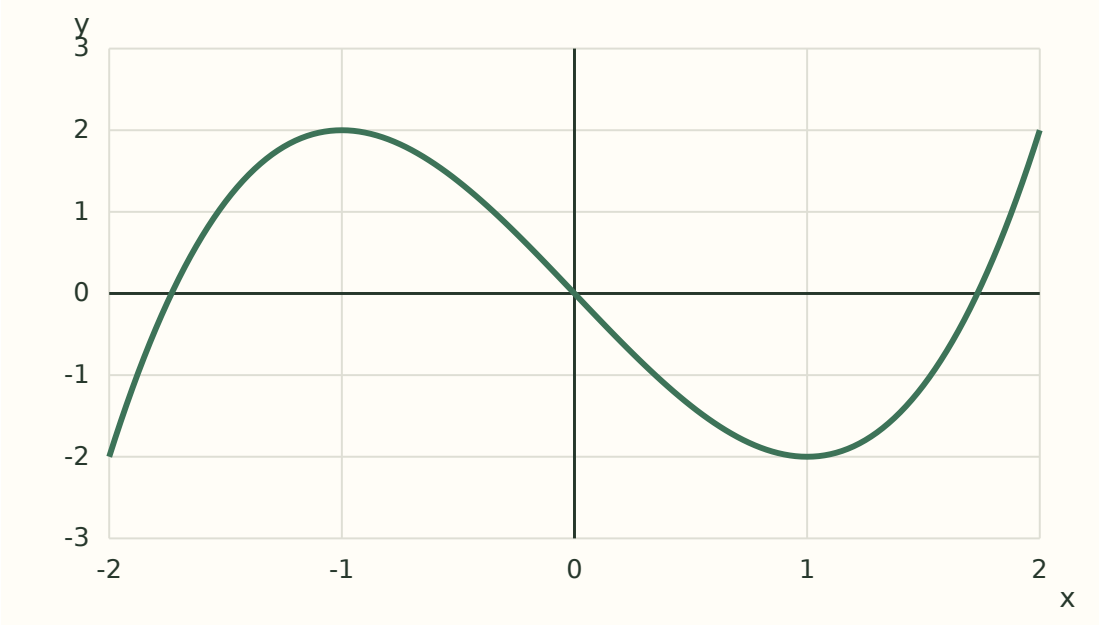
Name: _____ Date: _____

10 questions | 49 marks | 65 minutes

Analyse different representations, tangent conditions, changes of direction and the validity of solutions.

A scientific calculator may be used.

**1.** The supplied graph represents $f(x) = x^3 - 3x$. [5 marks]




**(a)** Read the approximate stationary coordinates from the graph. [2 marks]

**(b)** Verify them exactly with a derivative. [3 marks]

**2.** Compare $f(x) = x^2$ and $g(x) = 2x^2 + 3$. [4 marks]

**(a)** Find both derivative functions. [2 marks]

**(b)** Explain how the factor 2 and the constant 3 affect gradients. [2 marks]

**3.** $f(x) = x^3 + kx$. Its tangent at x=1 has gradient 5. [4 marks]

**(a)** Find k. [2 marks]

**(b)** Find the tangent equation at that point. [2 marks]

**4.** For $f(x) = x^3 - x$ at the origin, [4 marks]

**(a)** find the tangent equation. [2 marks]

**(b)** find the normal and verify perpendicularity. [2 marks]

**5.** Analyse $f(x) = x(x-2)^2$. [6 marks]

**(a)** Find intercepts and stationary coordinates. [4 marks]

**(b)** Classify the stationary points. [2 marks]

**6.** $s(t) = t^3 - 9t^2 + 24t$ metres for $0 \leq t \leq 6$. [6 marks]

**(a)** Find rest times and total distance. [4 marks]

**(b)** At t=3, find acceleration and decide whether the particle is at rest. [2 marks]

**7.** For $C(x) = x^2 + 2x + 3$, [4 marks]

**(a)** find its unrestricted minimum. [2 marks]

**(b)** find its minimum if only $0 \leq x \leq 4$ is allowed. [2 marks]

**8.** A rectangular enclosure has a single internal divider parallel to its width. Total fencing is 60m. Let width be x and length y. [6 marks]

**(a)** Form the area in terms of x alone. [3 marks]

**(b)** Find the greatest area and dimensions. [3 marks]

**9.** $s(t) = t^3 - 6t^2 + 12t$ metres on $0 \leq t \leq 4$ seconds. [5 marks]

**(a)** Show the particle is at rest once inside the interval, without turning. [3 marks]

**(b)** Find displacement and total distance. [2 marks]

**10.** A student uses $A = x(20 - x)$ for a rectangle and writes $A' = 20 - x$, obtaining x=20. [5 marks]

**(a)** Correct the derivative and the stationary input. [2 marks]

**(b)** Explain the geometric fault in the proposed x=20 answer. [3 marks]