Counting and statistics - Practice test 1

10 questions | 45 marks | 60 minutes

Begin with counting choices, then analyse a small survey and an introductory normal model. Show formulas and explain your choices.

Equivalent correct methods earn the corresponding marks.

1. A lunch menu offers 4 mains,3 sides and 2 drinks. Every combination is available. [4 marks]

(a) Count the complete meals. [2 marks]

$4\times3\times2=24$.

Answer: 24.

(b) One main is unavailable. Count the remaining complete meals. [2 marks]

$3\times3\times2=18$.

Answer: 18.

2. Five different posters are displayed in a row. [4 marks]

(a) Count all possible rows. [2 marks]

$5!=120$.

Answer: 120.

(b) A particular poster must be at the left end. Count the rows. [2 marks]

Fix that poster, then arrange the other 4 in $4!=24$ ways.

Answer: 24.

3. A group of 9 students chooses a captain, deputy and treasurer, all different people. [4 marks]

(a) Explain whether order matters. [2 marks]

Changing which person holds which role changes the outcome.

Answer: Yes: the three roles are distinct.

(b) Count the assignments. [2 marks]

9P3=9×8×7=504.

Answer: 504.

4. A library chooses 3 books from 8 different novels. [4 marks]

(a) Count the selections if no reading order is specified. [2 marks]

8C3=8!/(3!5!)=56.

Answer: 56.

(b) Count the selections containing one specified novel. [2 marks]

Include the specified book and choose 2 of the remaining 7:7C2=21.

Answer: 21.

5. Consider the expansion of $(x+3)^4$. [5 marks]

(a) Write the five Pascal coefficients. [2 marks]

Each interior coefficient is the sum of the two above it.

Answer: 1,4,6,4,1.

(b) Expand the expression. [3 marks]

Use descending powers of x and ascending powers of 3.

The terms are x^4,4x^3(3),6x^2(9),4x(27),81.

Answer: $x^4+12x^3+54x^2+108x+81$.

6. A teacher randomly selects 30 of 300 students and records journey time and number of bus transfers. [4 marks]

(a) Identify the population and sample. [2 marks]

The target is all 300; only 30 are observed.

Answer: Population:300 students; sample:the 30 selected.

(b) Classify both recorded variables. [2 marks]

Time is measured; transfers are counted.

Answer: Journey time is continuous numerical; number of transfers is discrete numerical.

7. Five observed daily sales totals are 3,4,4,5,14. [5 marks]

(a) Find the mean, median and mode. [3 marks]

The sum is 30, so mean=30/5=6.

The middle value and most frequent value are 4.

Answer: Mean 6; median 4; mode 4.

(b) Which centre better describes an ordinary day in this sample? Explain. [2 marks]

The unusually large 14 pulls the mean above the cluster 3-5.

Answer: Median 4.

8. The ordered data are 2,3,5,7,8,10,14. Use the median-of-halves convention, excluding the middle value. [5 marks]

(a) Find the five-number summary. [3 marks]

Median 7; lower half 2,3,5 has median 3; upper half 8,10,14 has median 10.

Answer: (2,3,7,10,14).

(b) Find the IQR and range. [2 marks]

$10-3=7; 14-2=12$.

Answer: IQR 7; range 12.

9. A sample is 4,6,8. [5 marks]

(a) Calculate the sample mean and sample standard deviation. [3 marks]

Squared deviations are 4,0,4, total 8.

Sample variance=8/(3−1)=4; s=2.

Answer: Mean 6; s=2.

(b) Every reading increases by 10. Find the new mean and s. [2 marks]

A translation changes the centre but not deviations or spread.

Answer: Mean 16; s=2.

10. Bottle volumes follow an approximately normal model with mean 500 mL and standard deviation 5 mL. [5 marks]

(a) Estimate the percentage between 490 and 510 mL using the empirical rule. [2 marks]

The bounds are mean±2 standard deviations.

Answer: About 95%.

(b) Estimate the expected number above 510 mL among 4,000 bottles. Explain the word expected. [3 marks]

The upper tail is about 2.5%.

4000×0.025=100; actual counts can vary.

Answer: About 100 bottles; it is an average prediction, not a guaranteed batch count.