Counting and statistics - Practice test 2

10 questions | 50 marks | 65 minutes

Apply restrictions, work from a frequency table and compare distributions. Normal cumulative values are supplied where needed.

Equivalent correct methods earn the corresponding marks.

1. A four-digit number is made from digits 0-6 without repetition. [5 marks]

(a) Count all such four-digit numbers. [3 marks]

The leading digit has 6 choices, then 6,5,4 choices remain.

$6\times6\times5\times4=720$.

Answer: 720.

(b) Count the numbers ending in 0. [2 marks]

Fix 0 at the end; the first 3 positions use distinct digits from 1-6.

$6\times5\times4=120$.

Answer: 120.

2. Six different people stand in a row; two named people must stand together. [5 marks]

(a) Count the rows. [3 marks]

Treat the named pair as one block:5! arrangements.

The pair has 2 internal orders:$5!\times2=240$.

Answer: 240.

(b) Count the rows if those two must instead occupy the two ends. [2 marks]

Choose which named person is at the left in 2 ways.

Arrange the other 4 in 4! ways:$2\times24=48$.

Answer: 48.

3. A panel of 3 is chosen from 6 adults and 4 students. [5 marks]

(a) Count panels with exactly 2 adults. [2 marks]

6C2×4C1=15×4=60.

Answer: 60.

(b) Count panels with at least 1 student. [3 marks]

10C3−6C3=120−20=100.

Answer: 100.

4. Consider $(2x-1)^5$. [5 marks]

(a) Find the coefficient of x^3. [3 marks]

Choose 3 brackets contributing 2x:5C3×2^3×(−1)^2=80.

Answer: 80.

(b) Find the sum of all coefficients. [2 marks]

Substitute x=1:(2−1)^5=1.

Answer: 1.

5. A school has 420 juniors and 280 seniors. It wants a sample of 50. [5 marks]

(a) Allocate a proportional stratified sample. [2 marks]

420/700×50=30;280/700×50=20.

Answer: 30 juniors and 20 seniors.

(b) Describe a valid selection method and one possible source of remaining bias. [3 marks]

Use enrolment lists as sampling frames.

Follow up consistently and report missing responses.

Answer: Randomly select within each group; non-response may still cause bias.

6. The table records the number of library visits per week in a sample. [5 marks]

Visits	0	1	2	3
Frequency	2	3	4	1

(a) Find the sample size and mean number of visits. [3 marks]

Frequency total=2+3+4+1=10.

Weighted total=0×2+1×3+2×4+3×1=14; mean 14/10=1.4.

Answer: n=10; mean 1.4 visits.

(b) Find the median and mode. [2 marks]

The fifth observation is 1 and the sixth is 2.

The most frequent value is 2.

Answer: Median 1.5; mode 2.

7. Group A has five-number summary(4,8,12,16,24); Group B has(6,10,15,22,30). [5 marks]

(a) Compare medians and IQRs with numerical evidence. [3 marks]

A IQR=16−8=8; B IQR=22−10=12.

Answer: B has a 3-unit higher median and a 4-unit larger IQR.

(b) Does the summary prove every B value exceeds every A value? Explain. [2 marks]

The intervals overlap and the summaries do not list each observation.

Answer: No.

8. A sample contains 2,2,6,6. [5 marks]

(a) Calculate its mean and sample standard deviation to 3 decimal places. [3 marks]

Squared deviations total 16.

s=sqrt(16/3)=2.309401...

Answer: Mean 4; s≈2.309.

(b) The readings are in metres. What unit belongs to the variance? [2 marks]

Squaring deviations squares the measurement unit.

Answer: Square metres.

9. A normal model has mean 100 and standard deviation 10. Supplied values:$\Phi(1.5)=0.93319$, $\Phi(-0.5)=0.30854$. [5 marks]

(a) Find $P(95<X<115)$. [3 marks]

Standardized bounds are-0.5 and 1.5.

$0.93319-0.30854=0.62465$.

Answer: 0.62465.

(b) Find the expected number in that interval among 800 items. [2 marks]

$800\times0.62465=499.72$.

Answer: 499.72, approximately 500 items.

10. A company models completion time as normal with mean 40 min and sd 6 min. The standard-normal 90th percentile is 1.28155. [5 marks]

(a) Find the model's 90th-percentile completion time. [3 marks]

$40+6\times1.28155=47.6893$.

Answer: Approximately 47.69 min.

(b) A manager says every job must finish within that time. Evaluate the claim. [2 marks]

A 90th percentile is not a maximum and model assumptions may also be imperfect.

Answer: Incorrect; the model puts about 10% above that time.