Exponential growth and decay

6 questions | 14 marks

Equivalent correct methods earn the corresponding marks.

1. For $P(t) = 1200e^{0.05t}$ people, find $P'(0)$. [2 marks]

Differentiate: $P' = 60e^{0.05t}$.

At zero the exponential is 1.

Answer: 60 people per time unit

2. A mass is $M(t) = 50e^{-0.3t}$ grams. Express $M'(t)$ in terms of $M(t)$. [2 marks]

The model has the form Ae^{kt}.

The derivative is k times the original quantity.

Answer: $M'(t) = -0.3M(t)$

3. Find the doubling time for $Q(t) = 10e^{0.4t}$. [2 marks]

Set $20 = 10e^{0.4t}$.

Take logs and solve for t.

Answer: $\ln 2/0.4 \approx 1.73$ time units

4. For $M(t) = 100e^{-0.2t}$ mg, find the rate when $M = 25$ mg. [2 marks]

Use $M' = -0.2M$.

Substitute $M = 25$.

Answer: -5 mg per time unit

5. Find the exact percentage increase in one year for $A(t) = 1000e^{0.03t}$. [3 marks]

The one-year multiplier is $e^{0.03}$.

Subtract 1 and multiply by 100.

Answer: $100(e^{0.03} - 1)\% \approx 3.045\%$

6. A cooling excess above room temperature is $E(t) = 40e^{-0.2t}$ degrees Celsius. Does this predict the actual temperature tends to 0 degrees Celsius? [3 marks]

Identify what E measures.

A zero excess means the object has reached room temperature.

Answer: No. Only the excess tends to zero; actual temperature tends to room temperature.