Inflection points

6 questions | 16 marks

Equivalent correct methods earn the corresponding marks.

1. Find the inflection point of $y = x^3 + 2x$. [2 marks]

The second derivative is $6x$.

It changes sign at zero; substitute into the original function.

Answer: $(0, 0)$

2. Explain why $y = x^4 + 1$ has no inflection point. [2 marks]

Zero at the origin is only a candidate.

The curve is concave up on both sides.

Answer: Its second derivative $12x^2$ never changes sign.

3. Find the inflection points of $y = x^4 - 4x^3$. [4 marks]

The second derivative is $12x(x - 2)$.

Its sign changes at both 0 and 2.

Substitute both values into the original function.

Answer: $(0, 0)$ and $(2, -16)$

4. Find the tangent at the inflection of $y = x^3 - 3x^2$. [3 marks]

The inflection is $(1, -2)$.

The derivative there is $3 - 6 = -3$.

Use $y + 2 = -3(x - 1)$.

Answer: $y = -3x + 1$

5. Does $y = e^x$ have an inflection? Justify. [2 marks]

Its second derivative is e^x.

This is positive everywhere, so concavity does not change.

Answer: No.

6. Classify the origin of $y = x^5$ as stationary/nonstationary and maximum/minimum/inflection. [3 marks]

The first derivative $5x^4$ is zero at 0.

The second derivative $20x^3$ changes sign.

The function continues increasing through zero.

Answer: Stationary inflection.