Stationary-point classification

6 questions | 15 marks

Equivalent correct methods earn the corresponding marks.

1. Find and classify the stationary point of $y = x^2 - 6x + 5$. [2 marks]

The derivative is $2x - 6$, zero at 3.

The second derivative is 2; the height is -4.

Answer: Local minimum $(3, -4)$.

2. Find and classify the stationary point of $y = -x^2 + 4x$. [2 marks]

The derivative is $-2x + 4$, zero at 2.

The second derivative is -2.

Answer: Local maximum $(2, 4)$.

3. Classify the stationary points of $y = x^3 - 3x^2$. [3 marks]

The derivative is $3x(x - 2)$.

The second derivative $6x - 6$ is negative at 0 and positive at 2.

Answer: Local maximum $(0, 0)$; local minimum $(2, -4)$.

4. Explain why the stationary point of $y = x^3$ is not an extremum. [2 marks]

The derivative is $3x^2$.

There is no increasing/decreasing reversal.

Answer: The derivative is positive on both sides of zero.

5. Classify the stationary point of $y = xe^{-x}$. [3 marks]

The derivative is $e^{-x}(1 - x)$.

Its sign changes from positive to negative at 1.

Answer: Local maximum $(1, e^{-1})$.

6. Can a local maximum fail to be an absolute maximum on an unbounded domain? Give an example. [3 marks]

A local maximum compares nearby inputs only.

As x grows, $x^3 - 3x$ grows without bound.

Answer: Yes: $x^3 - 3x$ has a local maximum at $(-1, 2)$ but is unbounded above.