Topic test | Differentiation and applications

Differentiation practice test 1

Name: _____ Date: _____

10 questions | 55 marks | 75 minutes

An original progressively demanding chapter paper. Show analytical working; a scientific calculator may be used for numerical values. Diagrams and data needed for questions are supplied.

A scientific calculator may be used.

1. Let $f(x) = 2x^3 - 3x^2 + 4$. [5 marks]

(a) Find $f'(x)$. [2 marks]

(b) Find the equation of the tangent at $x = 2$. [3 marks]

2. Let $g(x) = (x^2 + 1)(x - 2)$. [5 marks]

(a) Differentiate in product form, then simplify. [3 marks]

(b) Find the tangent at $x = 0$. [2 marks]

3. Let $h(x) = (2x - 1)^4$. [5 marks]

(a) Find the derivative. [2 marks]

(b) Find the stationary point and classify it. [3 marks]

4. Let $r(x) = (x^2 + 1)/(x + 1)$. [5 marks]

(a) State the domain and find the derivative. [3 marks]

(b) Find the gradient at $x = 1$. [2 marks]

5. Consider $y = 3e^{2x}$. [4 marks]

(a) Find the derivative. [2 marks]

(b) Find the tangent at the y-intercept. [2 marks]

6. For radian input, $f(x) = 2\sin(3x) - \cos x$. [4 marks]

(a) Differentiate. [2 marks]

(b) Find the tangent at $x = 0$. [2 marks]

7. Analyse $F(x) = x^3 - 6x^2 + 9x$. [7 marks]

(a) Find and classify both stationary points. [4 marks]

(b) Find its inflection point, justifying the change. [3 marks]

8. A medicine model is $M(t) = 200e^{-0.05t}$ mg, with t in hours. [6 marks]

(a) Find the rate at $t = 10$, to 3 significant figures. [3 marks]

(b) Find its half-life. [3 marks]

9. A rectangle has perimeter 40 cm. Let one side be x cm. [5 marks]

(a) Form an area model and state the physical domain. [2 marks]

(b) Find and justify the greatest area. [3 marks]

10. A particle moves on a line with $s(t) = t^3 - 9t^2 + 24t$ metres for $0 \leq t \leq 6$ seconds. [9 marks]

(a) Find the rest times and the interval of negative motion. [3 marks]

(b) Find displacement and total distance. [4 marks]

(c) Is the particle speeding up at 5 seconds? [2 marks]